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Characterization of a liquid bath for accurate thermometer calibration and detailed uncertainty estimation

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ABSTRACT
The reliable characterization of liquid baths is essential for ensuring traceable and accurate temperature calibrations in metrology laboratories. In this study, a liquid water bath with a copper equalizing block of 12 cm length was characterized at three calibration points of 5 °C, 29.7 °C, and 80 °C. The bath performance was assessed through drift evaluation, stability, axial and radial uniformity, and loading influence using two 25 Ohm (Ω) standard platinum resistance thermometers (SPRTs). The results obtained showed excellent drift performance within ± 1 millikelvins (mK) over 12 hours at 20 °C. The instability at the three calibration points remained below 1 mK. Axial and radial uniformity values did not exceed 0.9 m 2.7 mK, respectively, and the loading effect produced temperature deviations of less than 3 mK. A detailed uncertainty analysis was carried out based on the requirements of the *Guide to the expression of uncertainty in measurement* (ISO GUM), considering all relevant contributions from the SPRTs and the liquid bath. The combined standard uncertainties of the bath characterization were 2.70, 3.17, and 3.87 mK at 5 °C, 29.7 °C, and 80 °C, respectively, all well below the 10 mK benchmark typically required for precision calibration baths. After the liquid bath characterization, calibration of a platinum resistance thermometer (PRT) by comparison against a 100-Ω SPRT was performed at 5 °C, 29.7 °C, and 80 °C. The associated expanded uncertainties of calibration were 0.052 °C, 0.050 °C, and 0.043 °C, respectively, including the bath characterization uncertainty. These findings confirm that the characterized liquid bath provides a stable and homogeneous thermal environment suitable for high-accuracy thermometer calibrations.

Section: RESEARCH PAPER

Keywords: liquid bath; uniformity; stability; thermometer calibration; uncertainty

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1. INTRODUCTION

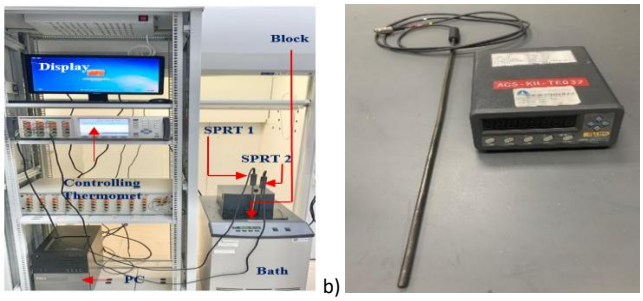
The characterization of liquid calibration baths plays a crucial role in the quality of thermal calibrations, because liquid baths provide stable and homogeneous thermal environments for accurate calibration of thermometers by comparison methods [1]. The metrologically sound evaluation of the stability and uniformity of liquid baths is an important research concern. Such evaluations help facilitate the reproducibility of the liquid bath systems and enhance their ability to improve calibration accuracy [2]. There are various types of liquid baths, each suited to specific metrological applications depending on the temperature range and properties of the liquids used. For instance, stirred liquid calibration baths can operate across a wide temperature range from -50 °C to +600 °C depending on the type of liquid used [3]–[6]. However, issues such as bubble formation and changes in fluid behaviour at high temperatures have limited testing to narrower ranges, such as 150 °C to 250 °C, in some silicone oil baths [7]. The thermal stability of a liquid bath is defined as the maximum temperature difference at a fixed set point over a specified period of time. On the other hand, uniformity is defined as the maximum temperature variation between measurement points within the bath [8]. It is worth noting that few published studies exist on the evaluation of the stability and uniformity of liquid baths, and the uncertainty associated with these assessments. Andrea et al. at the Italian National Institute of Metrology (IMGC) have designed, built, and described a new liquid oil bath intended to provide a suitable apparatus for thermometer calibration at the millikelvin (mK) level over the range of -10 °C to 100 °C [9]. They concluded that the bath temperature in the measurement zone remained stable within few millikelvins over several days, and was unaffected by ambient temperature fluctuations. They also reported that the bath

exhibited excellent radial and axial temperature uniformity within a few millikelvins across a 10 cm central portion of the measurement zone. Pagtalunan et al. reported a method for characterizing a liquid bath using silicone oil over the range of 90 °C to 250 °C. They used two standard platinum resistance thermometers (SPRTs) connected to a digital readout device to take measurements at various locations and immersion depths throughout the working volume of the calibration bath [4]. They found that the bath was stable at lower temperatures with improving stability as the temperature approached 200 °C. They also noted that the bath exhibited better uniformity at lower temperature settings. Dekic et al. investigated the uniformity of a cylindrical calibration bath [10]. They assessed the bath uniformity by measuring axial and radial temperature gradients using SPRTs, one fixed and the other mobile. They evaluated stability by observing temperature fluctuations over a specified time period. Their conclusion was that, when the results are compared with the manufacturer specifications, they can be used to determine the contribution of the calibration bath to the overall measurement uncertainty during thermometer calibration. In this paper, a water bath was characterized by examining its drift at 20 °C for 12 hours first, followed by examining stability for one hour at three calibration points, 5 °C, 29.7 °C, and 80 °C [9], [11]. After that, radial and axial uniformity, as well as the effect of loading were investigated at each calibration temperature. The study was conducted using a copper block approximately 12 cm in length and two SPRTs of 25-Ohm, one fixed and the other movable. The associated uncertainty with the bath characterization results was estimated. Then, the characterized water bath was used for the calibration of a platinum resistance thermometer, PRT, by comparison against a standard 100-Ω SPRT at 5, 29.7, and 80 °C. A detailed uncertainty budget including the bath characterization and the PRT calibration was calculated based on the requirements of the ISO GUM and reported in this paper [12].

2. EXPERIMENTAL

2.1. Equipment

The bath used to maintain the triple point of water was the model 7312, manufactured by Fluke, USA. Its liquid capacity is 19 L and the working volume is 5800 cm^3, i.e 5.8 L filled with distilled water. A copper block, of 120 mm length, supplied by Fluke was used to accommodate the temperature sensors. It has five bores, each is 8 mm in diameter. Temperature measurements in the measurement zone were carried out using two long-stem 25-Ω standard platinum resistance thermometers (SPRTs), model 5680, manufactured by Hart Scientific, USA. The maximum length of the sensor is 5 mm and the outside diameter is 6 mm. During measurements, one SPRT remained fixed, while the other was movable. The control thermometer, model 1595A, offering a resolution of 0.00001 °C was supplied by Fluke, USA. The equipment setup is depicted in Figure 1, which shows the liquid bath characterization (a) and the PRT under calibration (b).



a) b)

Figure 1. The calibration setup of the liquid bath characterization (a) and the PRT under calibration (b).

The PTR under calibration was a product of Fluke, model 5628, of 6.35 mm diameter, 381 mm length, 0.001 resolution, and temperature range of -200 to 661 °C. In the meanwhile, the calibration standard was a 100-Ω SPRT, model 5682, manufactured by Fluke, USA.

2.2. Drift of the liquid bath

To assess the drift in temperature over time, the bath was set to 20 °C, and after stabilization, the temperature was continuously recorded by an SPRT submerged at the 0-cm depth for 12 hours at a reading every 30 seconds [9]. During this period, the system was under the control of a LABVIEW program. The mean of every 8 readings (taken in 4 minutes) was calculated and this gives a total of 15 data points per hour.

2.3. Stability of the liquid bath

The stability of the water bath was evaluated at each calibration temperature (5 °C, 29.7 °C, and 90 °C). The reference SPRT was submerged in one of the block bores at 0-cm depth and connected to the control thermometer. The bath temperature was set to 5 °C and allowed to stabilize for two hours. Once the thermal stability of the bath was reached, temperature readings were recorded for one hour at 30 second intervals [13]. The same procedure was repeated at 29.7 and 80 °C.

2.4. Radial uniformity of the liquid bath

The radial uniformity of the liquid bath was assessed using both (SPRTs). They were submerged in two radially opposite bores of the calibration block at the same 0-cm depth. After thermal stability at 5 °C, temperature readings were taken for one hour at intervals of 30 seconds [13]. The thermometers were then repositioned to a different pair of radially opposite bores, and another set of temperature readings were collected under the same conditions. The same procedure was repeated at 29.7 and 80 °C.

2.5. Axial uniformity of the liquid bath

The axial uniformity of the bath was assessed using the two SPRTs. The reference SPRT was fixed at 0-cm depth throughout the experiment. The mobile SPRT was initially fixed at the same 0-cm depth, and once thermal stability was achieved, both thermometers recorded temperature readings for 10 minutes at 30-second intervals, and the average was calculated [13]. Then, the second SPRT was moved up from the bottom by 2 cm. After stability was reached, temperature was recorded for another 10 minutes at intervals of 30 seconds. Finally, the mobile SPRT was moved up further to 4 cm, and after stability, temperature was recorded for another 10 minutes at intervals of 30 seconds. After that, the moving SPRT was submerged at 0-cm depth again to make sure that the initial measurement is correct. This procedure was followed at 5 °C, 29.7 °C, and 80 °C.

2.6. The loading effect

To evaluate the effect of thermal loading, a baseline measurement was first recorded with only the reference SPRT inserted in one of the bores at 0-cm depth, and the other bores of the block were left empty. The bath was allowed to stabilize at 5 °C, and temperature readings were recorded over a period

of 15 minutes at intervals of 30 seconds. Then, multiple Pt100 sensors were inserted in the empty bores of the block to simulate a typical calibration load [14]. After reaching thermal equilibrium, the temperature of the SPRT was recorded again under loaded conditions over a period of 15 minutes at intervals of 30 seconds to assess any deviations. This procedure was applied at 5, 29.7, and 80 °C.

2.7. Calibration of the Gust thermometer (PRT) by comparison

Calibration of the PRT was performed in the characterized liquid bath by comparison against a calibrated reference 100-Ω SPRT. The PRT under calibration was inserted into one of the block bores at a depth of 0 cm, and the SPRT was placed in an adjacent bore at the same 0-cm depth with a separation of less than 1 cm. The SPRT readings were monitored using the super thermometer, while the PRT readings were recorded on its own readout shown in Figure 1(b). A probe holder was used to keep both thermometers stable and vertically aligned throughout the measurement. This arrangement ensured reliable and repeatable comparison of temperature readings under well-controlled thermal conditions. The calibration was performed by setting the bath temperature at 5 °C, and after thermal equilibrium, 20 temperature readings were recorded at intervals of 30 seconds. The same calibration procedure was followed at 29.7 and 80 °C.

3. RESULTS AND DISCUSSION

3.1. Drift of the liquid bath temperature

Figure 2 presents the relation between the recorded 15 temperatures reading/hour over the 12-hour period. In this period of study, the highest recorded temperature was 19.91611 °C and the lowest was 19.91520 °C, giving a T_{max} - T_{min} of 0.00091 °C or 0.91 mK. This remarkably small fluctuation indicates that the liquid bath temperature fluctuations at room temperature were stable within ± 1 mK throughout the entire acquisition period, which is in agreement with the conclusion reported by Andrea and his coauthors [9]. Such a high level of stability implies that the system can maintain its set temperature within this narrow margin, regardless of any fluctuations in the ambient environmental conditions. This performance demonstrates that the bath meets the criterion of thermal stability, making it suitable for stability, uniformity, and load response studies. It is worth noting that the bath ability to maintain temperature within ± 1 mK is critical in metrology, materials testing and sensor calibration, where even minor thermal deviations can impact measurement accuracy or material behaviour.

Table 1. The temperature instability results of the liquid bath at 5 °C, 29.7 °C, and 80 °C.

	5 °C	29.7 °C	80 °C
Measured temperature	5.05583	29.44438	79.77740
	5.05590	29.44440	79.77733
	5.05600	29.44426	79.77764
	5.05594	29.44418	79.77765
	5.05591	29.44411	79.77765
	5.05581	29.44410	79.77775
	5.05573	29.44404	79.77697
	5.05581	29.44397	79.77801
	5.05576	29.44399	79.77794
	5.05578	29.44395	79.77784
	5.05579	29.44401	79.77789
	5.05573	29.44395	79.77794
	5.05574	29.44398	79.77779
	5.05576	29.44386	79.77757
	5.05574	29.44376	79.77751
T_{max}	5.05600	29.44440	79.77801
T_{min}	5.05573	29.44376	79.77697
$(T_{max}-T_{min})/2$	0.00014	0.00032	0.00052

3.2. Stability of the bath

The thermal stability of the liquid bath using the two 25-Ω SPRTs submerged into the equalizing copper block at 0-cm depth was carried out as described above. The temperature readings of both SPRTs were recorded over 60 minutes at 30-second intervals, and the average ($\bar{x}$) of the readings was calculated every 4 minutes to produce 15 data points for each thermometer at 5, 29.7, and 80 °C. Then, the mean of the two SPRTs' readings was calculated for each of the 15 points. These results are reported in Table 1. The T_{max} and the T_{min} were identified at each calibration temperature, and the thermal instability of the liquid bath was quantified as $(T_{Max} - T_{Min})/2$ as shown in Table 1. From this table, it can be seen that temperature instabilities of the liquid bath at 5 °C, 29.7 °C, and 80 °C were found to be 0.000135 °C, 0.000322 °C, and 0.000779 °C, i.e. 0.135, 0.322, and 0.779 mK, respectively. Thus, the instability of the liquid bath does not exceed 3 mK, which assures the validity of the bath for thermometer calibration.

3.3. Radial uniformity

Radial (horizontal) non-uniformity describes the temperature variation between different radial positions within the measurement zone of the liquid bath. To evaluate this effect, two

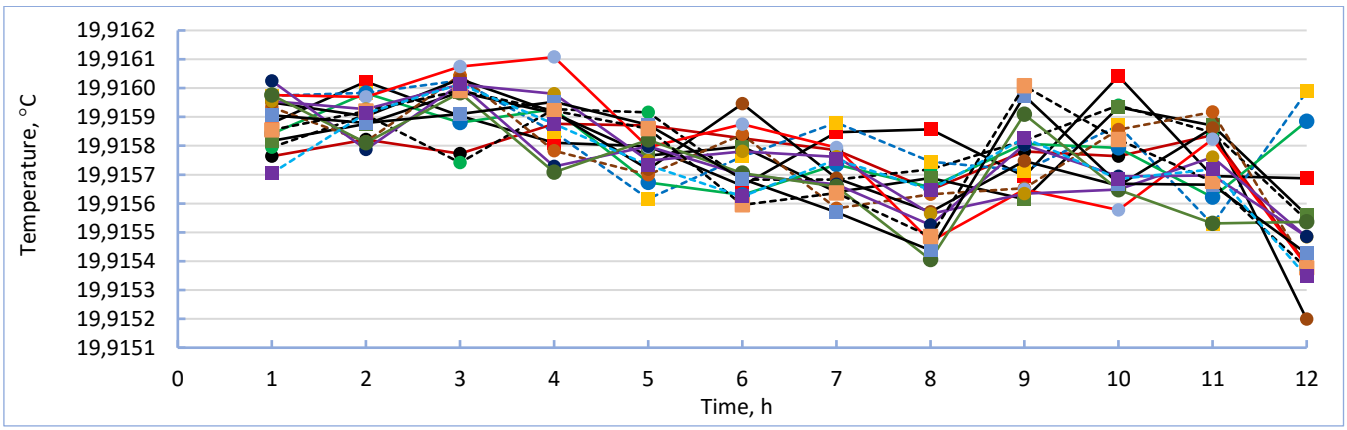


Figure 2. The results of the liquid bath drift at 20 °C for 12 h.

Table 2. The temperature results of the bath radial uniformity at 5 °C, 29.7 °C, and 80 °C.

	5 °C		29.7 °C		80 °C	
	Radial 1	**Radial 2**	**Radial 1**	**Radial 2**	**Radial 1**	**Radial 2**
Measured temperature	5.05744	5.05717	29.44389	29.44409	79.77973	79.77898
	5.05750	5.05727	29.44378	29.44397	79.78004	79.77949
	5.05736	5.05725	29.44350	29.44407	79.78009	79.77936
	5.05720	5.05720	29.44363	29.44426	79.77951	79.77938
	5.05728	5.05730	29.44386	29.44464	79.77873	79.77899
	5.05724	5.05726	29.44410	29.44465	79.77857	79.77862
	5.05711	5.05723	29.44372	29.44439	79.77765	79.77793
	5.05710	5.05728	29.44416	29.44437	79.78037	79.77806
	5.05721	5.05722	29.44395	29.44428	79.78004	79.77821
	5.05729	5.05732	29.44386	29.44429	79.77966	79.77850
	5.05716	5.05729	29.44390	29.44439	79.77951	79.77876
	5.05712	5.05735	29.44390	29.44449	79.77959	79.77919
	5.05703	5.05727	29.44387	29.44450	79.77985	79.77937
	5.05702	5.05728	29.44392	29.44453	79.78025	79.77958
	5.05710	5.05723	29.44375	29.44443	79.78036	79.77938
T_{max}	5.05750	5.05735	29.44416	29.44465	79.78037	79.77958
T_{min}	5.05702	5.05717	29.44350	29.44397	79.77765	79.77793
T_{max} -T_{min}	0.00048	0.00018	0.00067	0.00069	0.00272	0.00165

SPRTs were placed in diametrically opposite bores of the block at a depth of 0 cm. After thermal equilibrium was reached, measurements were performed at the calibration temperatures of 5 °C, 29.7 °C, and 80 °C. At each temperature point, the SPRTs were monitored over 30 minutes, with a reading taken every 30 s. These readings were averaged over 2 min intervals, yielding 15 averaged values. The resulting dataset was recorded in Table 2 as radial 1. The two thermometers were then repositioned into a different pair of diametrically opposite bores, and the same procedure was repeated to obtain a second dataset as radial 2.

For each dataset, the maximum and minimum temperatures were identified and the difference ($T_{max} - T_{min}$) was calculated. The largest observed differences, which represent the radial non-uniformity of the block, were 0.00048 °C, 0.00069 °C, and 0.0027 °C (0.48 mK, 0.69 mK, and 2.70 mK) at 5 °C, 29.7 °C, and 80 °C, respectively. These results demonstrate that the liquid bath maintains a very high degree of radial uniformity in the measurement zone across the tested temperatures. Even at the highest calibration temperature, 80 °C, the maximum observed non-uniformity was only 2.70 mK, which is small enough compared with the typical uncertainties of precision thermometer calibrations. Therefore, the liquid bath provides highly stable and uniform thermal conditions in the measurement zone, ensuring reliable traceability and accuracy in calibration measurements.

3.4. Axial uniformity

Axial uniformity of the temperature within the water bath was evaluated using the two SPRTs. They were immersed vertically in the equalizing block at different depths: 0 cm, 2 cm, and 4 cm, to assess the vertical (axial) temperature distribution within the measurement zone. Measurements were performed at the three calibration temperatures: 5 °C, 29 °C, and 80 °C. At each point, the two SPRTs were fixed at the specified depth, and the system was allowed to reach thermal equilibrium. Once stability was achieved, temperature readings were recorded over 30 minutes at 30-second intervals, and the mean of each 4 readings was calculated to obtain 15 measurement points. The mean ($\bar{x}$) of the 15 measurements at each immersion depth, 0 cm, 2 cm, and 4 cm, was calculated and reported in Table 3. The temperature difference ($\bar{x}_{Ai}$ - $\bar{x}_0$) between the mean at 2 cm and 4 cm ($\bar{x}_{Ai}$) and the mean ($\bar{x}_0$) at 0 cm was calculated at 5 °C, 30 °C, and 80 °C according to equation (5), and reported in Table 3. These differences indicate how uniformly temperature is distributed along the vertical axis within the equalizing block. The largest difference at each calibration point was taken to represent the axial uniformity of the temperature in the measurement zone. It can be realized that the temperature variations at the three immersion depths did not exceed 0.9 mK,

Table 3. The temperature results of the bath axial uniformity at 5 °C, 29.7 °C, and 80 °C.

	5 °C			29.7 °C			80 °C		
	Axial 0	**Axial 2**	**Axial 4**	**Axial 0**	**Axial 2**	**Axial 4**	**Axial 0**	**Axial 2**	**Axial 4**
Measured temperature	5.05694	5.05676	5.05660	29.44417	29.44377	29.44419	79.77576	79.77555	79.77465
	5.05697	5.05685	5.05656	29.44410	29.44385	29.44417	79.77612	79.77516	79.77468
	5.05711	5.05687	5.05660	29.44409	29.44389	29.44414	79.77652	79.77519	79.77451
	5.05700	5.05675	5.05651	29.44407	29.44399	29.44421	79.77686	79.77544	79.77428
	5.05708	5.05677	5.05654	29.44378	29.44396	29.44426	79.77689	79.77578	79.77413
	5.05716	5.05610	5.05682	29.44376	29.44378	29.44423	79.77687	79.77587	79.77451
	5.05716	5.05705	5.05686	29.44376	29.44372	29.44413	79.77679	79.77499	79.77514
	5.05705	5.05683	5.05658	29.44380	29.44373	29.44419	79.77656	79.77554	79.77504
	5.05690	5.05678	5.05667	29.44393	29.44394	29.44403	79.77658	79.77552	79.77556
	5.05688	5.05683	5.05677	29.44365	29.44400	29.44400	79.77662	79.77571	79.77587
	5.05687	5.05682	5.05674	29.44391	29.44393	29.44402	79.77652	79.77550	79.77608
	5.05686	5.05690	5.05666	29.44417	29.44386	29.44413	79.77664	79.77521	79.77637
	5.05677	5.05695	5.05665	29.44437	29.44394	29.44408	79.77667	79.77517	79.77640
	5.05666	5.05700	5.05673	29.44415	29.44388	29.44414	79.77665	79.77558	79.77619
	5.05667	5.05697	5.05676	29.44413	29.44394	29.44411	79.77657	79.77565	79.77601
$\bar{x}$	5.05694	5.05687	5.05667	29.44399	29.44388	29.44413	79.77657	79.77546	79.77530
$\bar{x}_{Ai}$ - $\bar{x}_0$	0.00000	-0.00007	-0.00027	0.00000	-0.00011	0.00015	0.00000	-0.00112	-0.00118

Table 4. temperature results of the bath loading at 5 °C, 29.7 °C, and 80 °C.

	5 °C		29.7 °C		80 °C	
	Axial 0 (A_0)	Loading (L)	Axial 0 (A_0)	Loading (L)	Axial 0 (A_0)	Loading (L)
Measured temperature	5.05694	5.05784	29.44417	29.44432	79.77576	79.78030
	5.05697	5.05761	29.44409	29.44435	79.77612	79.78013
	5.05711	5.05741	29.44409	29.44448	79.77652	79.78012
	5.05700	5.05742	29.44407	29.44458	79.77686	79.77991
	5.05708	5.05711	29.44378	29.44457	79.77689	79.77921
	5.05716	5.05703	29.44375	29.44447	79.77687	79.77885
	5.05716	5.05712	29.44376	29.44356	79.77679	79.77785
	5.05705	5.05722	29.44380	29.444301	79.77656	79.77937
	5.05690	5.05724	29.44393	29.44441	79.77658	79.77920
	5.05688	5.05706	29.44364	29.44459	79.77662	79.77908
	5.05687	5.05725	29.44391	29.44465	79.77652	79.77882
	5.05686	5.05720	29.44417	29.44462	79.77664	79.77835
	5.05677	5.05692	29.44437	29.44467	79.77667	79.77852
	5.05666	5.05681	29.44415	29.44457	79.77665	79.77884
	5.05667	5.05677	29.44413	29.44446	79.77657	79.77929
$\bar{x}$	5.05694	5.05720	29.44399	29.44444	79.77657	79.77919
$\bar{x}_{A0}$- $\bar{x}_L$		0.00026		-0.00045		0.00260

demonstrating a high level of axial uniformity in the bath under the tested conditions.

3.5. Effect of loading

The effect of loading on temperature uniformity within the measurement zone was studied by placing the SPRT thermometer in one of the block bores at a depth of 0 cm, while several Pt100 sensors were placed in the remaining bores. The bath temperature was then set at each calibration point until thermal stability was reached. Readings from the SPRT were recorded for 15 minutes at intervals of 30 seconds, and the mean of every two consecutive readings was calculated to obtain 15 measurement points, as shown in Table 4. The mean of these 15 points ($\bar{x}_L$) was then calculated and subtracted from the mean of another 15 readings obtained during the axial uniformity study ($\bar{x}_{A0}$) at the same depth of 0 cm. The correction due to the loading effect ($\bar{x}_{A0}$- $\bar{x}_L$) was found to be 0.00026 °C, 0.00045 °C, and 0.0026 °C (i.e 0.26 mK, 0.45 mK, and 2.6 mK) at 5 °C, 29.7 °C, and 80 °C, respectively. It is to be noted that these values are small enough and consistant with the DKD-R 5-4 conclusion that, though the effect of loading of the liquid bath exists, it is much smaller than in the case of dry block [14].

3.6. Uncertainty of the liquid bath characterization

Evaluation of the uncertainty arising from the characterization of a liquid bath used constitutes an essential component of the overall uncertainty associated with the thermometer calibration results [13], [14]. In accordance with ISO/IEC 17025 requirements, calibration laboratories must account for all potential sources of uncertainty. Examination of the bath characterization procedure indicates that the contributing uncertainty sources include: calibration, drift, self-heating of the SPRTs, calibration and resolution of the super thermometer, bath radial and axial uniformity, stability, and the loading effect. The mathematical model governing this uncertainty estimation is shown in equation (1), and the uncertainty sources are summarized in the fishbone diagram presented in Figure 3. Estimation of the uncertainty of these contributions was carried out in accordance with ISO GUM as detailed below.

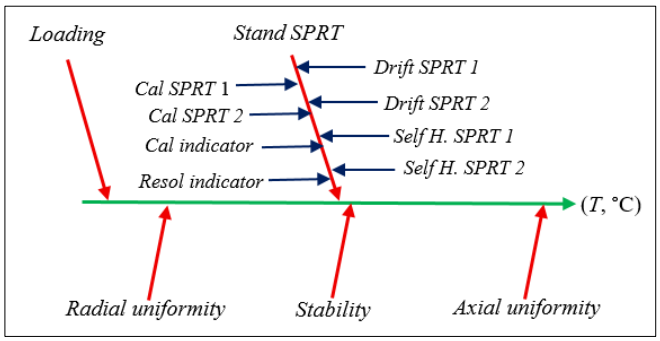


Figure 3. Fishbone showing the uncertainty sources of the liquid bath characterization.

$$\delta t = \delta t_{\text{stand}} + \delta t_{\text{uniform}} + \delta t_{\text{instab}} + \delta t_{\text{load}} \,. \quad (1)$$

3.7. Uncertainty of the standard SPRTs

To estimate the uncertainty arising from the seven sources associated with the SPRTs, as shown in Figure 3, the calibration uncertainty was divided by 2, while the drift and self-heating uncertainties were divided by $\sqrt{3}$ assuming a rectangular distribution. The calibration uncertainties of both thermometers were combined using the formula: ${u_{\text{cal}}}^2 = (Cal\ SPRT\ 1)^2 + (Cal\ SPRT\ 2)^2$. Similarly, the drift and the self heating uncertainties of the two SPRTs were combined according to the formulas: ${u_{\text{Drift}}}^2 = (Drift\ SPRT\ 1)^2 + (Drift\ SPRT\ 2)^2$ and ${u_{\text{Self H}}}^2 = (Self\ H\ SPRT\ 1)^2 + (Self\ H\ SPRT\ 2)^2$, respectively. Moreover, the uncertainty arising from the super thermometer (indicator) comes from the calibration and resolution. Since the uncertainty of the calibration is expressed in Ohm, a sensitivity coefficient c_4 of 0.385 Ω was applied to express the uncertainty in units of temperature. In addition, the uncertainty due to the resolution was calculated using equation (2):

$$u_{\text{Resol}} = \frac{Resolution}{2\sqrt{3}} \,. \quad (2)$$

Thus, the uncertainty of the indicator thermometer was expressed in the formula ${u_{\text{Indicator}}}^2 = {u_{\text{Cal Indicator}}}^2 + {u_{\text{Resol Indicator}}}^2$. Then, the combined standard uncertainty associated with the SPRTs, u_{stand}, was then calculated according to equation (3), in which each of the sensitivity coefficients c_1, c_2, and c_3 equals 1, since the uncertainty was expressed in units of temperature [11], [15]. The calculated uncertainty values at the calibration points 5 °C, 29.7 °C, and 80 °C are shown in Table 5, Table 6, and Table 7, respectively.

$$u_{\text{stand}} = \sqrt{(c_1 \cdot u_{\text{Cal}})^2 + (c_2 \cdot u_{\text{Drift}})^2 + (c_3 \cdot u_{\text{Self H}})^2 + (c_4 \cdot u_{\text{Indicator}})^2} \,. \quad (3)$$

Table 5. The uncertainty budget of the bath characterization at 5 °C.

Uncertainty Source	Description	Estimate	u_{xi}	Unit	Distribution	Divisor	c_i	$c_i \cdot u_{xi}$
	Cal SPRT 1	0	0.39	mK	Normal	2	1	0.3850
	Cal SPRT 2	0	1.10	mK	Normal	2	1	1.1000
	Drift SPRT 1	0	1.73	mK	Rectangular	$\sqrt{3}$	1	1.7321
	Drift SPRT 2	0	1.67	mK	Rectangular	$\sqrt{3}$	1	1.6743
Standard SPRTs	Cal indicator	0	0.000050	Ω	Normal	2	0.385	0.019
	Resol indicator	0	0.00288675	mK	Rectangular	$\sqrt{3}$	1	0.003
	Self H. SPRT 1	0	0.02309	mK	Rectangular	$\sqrt{3}$	1	0.02309
	Self H. SPRT 2	0	0.02194	mK	Rectangular	$\sqrt{3}$	1	0.02194
							u_{stand}	2.68
Radial uniformity	$\delta t_{uniformity}$	0	0.27540	mK	Rectangular	$\sqrt{3}$	1	0.27540
Axial uniformity		0	0.15563	mK	Rectangular	$\sqrt{3}$	1	0.15563
Loading effect	δt_{Load}	0	0.15163	mK	Rectangular	$\sqrt{3}$	1	0.15163
Stability in time	δt_{instab}	0	0.07809	mK	Rectangular	$\sqrt{3}$	1	0.07809
Combined standard uncertainty (bath characterization), u_c				mK				2.70

Table 6. The uncertainty budget of the bath characterization at 29.7 °C.

Uncertainty Source	Description	Estimate	u_{xi}	Unit	Distribution	Divisor	c_i	$c_i \cdot u_{xi}$
	Cal SPRT 1	0	0.39	mK	Normal	2	1	0.3850
	Cal SPRT 2	0	1.10	mK	Normal	2	1	1.1000
	Drift SPRT 1	0	1.73	mK	Rectangular	√3	1	1.7321
	Drift SPRT 2	0	1.67	mK	Rectangular	√3	1	1.6743
Standard SPRTs	Cal indicator	0	0.000050	Ω	Normal	2	0.385	0.019
	Resol indicator	0	0.00288675	mK	Rectangular	√3	1	0.003
	Self H. SPRT 1	0	0.02309	mK	Rectangular	√3	1	0.02309
	Self H. SPRT 2	0	0.02194	mK	Rectangular	√3	1	0.02194
							u_{stand}	2.68
Radial uniformity	$\delta t_{uniformity}$	0	0.39606	mK	Rectangular	√3	1	0.39606
Axial uniformity		0	0.08370	mK	Rectangular	√3	1	0.08370
Loading effect	δt_{Load}	0	0.26064	mK	Rectangular	√3	1	0.26064
Stability in time	δt_{instab}	0	0.18591	mK	Rectangular	√3	1	0.18591
Combined standard uncertainty (bath characterization), u_c				mK				3.17

Table 7. The uncertainty budget of the bath characterization at 80 °C.

Uncertainty Source	Description	Estimate	u_{xi}	Unit	Distribution	Divisor	c_i	$c_i \cdot u_{xi}$
	Cal SPRT 1	0	0.39	mK	Normal	2	1	0.3850
	Cal SPRT 2	0	1.10	mK	Normal	2	1	1.1000
	Drift SPRT 1	0	1.73	mK	Rectangular	$\sqrt{3}$	1	1.7321
	Drift SPRT 2	0	1.67	mK	Rectangular	$\sqrt{3}$	1	1.6743
Standard SPRTs	Cal indicator	0	0.000050	Ω	Normal	2	0.385	0.019
	Resol indicator	0	0.00288675	mK	Rectangular	$\sqrt{3}$	1	0.003
	Self H. SPRT 1	0	0.02309	mK	Rectangular	$\sqrt{3}$	1	0.02309
	Self H. SPRT 2	0	0.02194	mK	Rectangular	$\sqrt{3}$	1	0.02194
							u_{stand}	2.68
Radial uniformity	$\delta t_{uniformity}$	0	1.56982	mK	Rectangular	$\sqrt{3}$	1	1.56982
Axial uniformity		0	0.73830	mK	Rectangular	$\sqrt{3}$	1	0.73830
Loading effect	δt_{Load}	0	1.51023	mK	Rectangular	$\sqrt{3}$	1	1.51023
Stability in time	δt_{instab}	0	0.30174	mK	Rectangular	$\sqrt{3}$	1	0.30174
Combined standard uncertainty (bath characterization), u_c				mK				3.87

3.8. Uncertainty due to the radial uniformity

The uncertainty due to the radial uniformity, u_{radial}, of temperature in the measurement zone at each calibration temperature was calculated by taking the largest $T_{\text{Max}} - T_{\text{Min}}$ whether it is within the data of Radial 1 or Radial 2 shown in Table 2. The $T_{\text{Max}} - T_{\text{Min}}$ was divided by $\sqrt{3}$ assuming a rectangular distribution in accordance with equation (4) [13]:

$$u_{\text{radial}} = \frac{T_{\text{Max}} - T_{\text{Min}}}{\sqrt{3}}. \quad (4)$$

The resulting uncertainty values at the calibration temperatures 5 °C, 29.7 °C, and 80 °C were reported in Table 5, Table 6, and Table 7, respectively.

3.9. Uncertainty of the axial uniformity

The uncertainty due to the axial uniformity of temperature in the measurement zone was calculated by taking the largest ($\bar{x}_{\mathrm{Ai}} - \bar{x}_0$) at each calibration point, as shown in Table 3. Such a maximum difference was then divided by $\sqrt{3}$ assuming a rectangular distribution, using equation (5) [13]:

$$u_{\mathrm{axial}} = \frac{\max\left|\bar{x}_{\mathrm{Ai}} - \bar{x}_0\right|}{\sqrt{3}}. \tag{5}$$

The resulting uncertainty values at the calibration temperatures 5 °C, 29.7 °C, and 80 °C were reported in Table 5, Table 6, and Table 7, respectively.

3.10. Uncertainty due to temperature instability

The uncertainty due to the temperature variations during the measuring cycle of 30 minutes was estimated as the half difference between the maximum and the minimum temperature, $(T_{\mathrm{Max}} - T_{\mathrm{Min}})/2$ using the measurement data in Table 1. The temperature difference was divided by $\sqrt{3}$ assuming a rectangular distribution, in accordance with equation (6).

$$u_{\mathrm{instab}} = \frac{T_{\mathrm{Max}} - T_{\mathrm{Min}}}{2\sqrt{3}}. \tag{6}$$

The resulting uncertainty values at the calibration temperatures 5 °C, 29.7 °C, and 80 °C were reported in Table 5, Table 6, and Table 7, respectively.

3.11. Uncertainty due to the loading effect

The uncertainty due to the effect of maximum loading of the liquid bath within the measurement zone with other thermometers was calculated as the difference between the average ($\bar{x}_{\mathrm{A0}}$) temperature at axial o-cm depth measured by one SPRT and the average ($\bar{x}_{\mathrm{L}}$) temperature measured by the same SPRT at the same depth but under loading, as shown in Table 4. The calculated ($\bar{x}_{\mathrm{A0}} - \bar{x}_{\mathrm{L}}$) was divided by $\sqrt{3}$, assuming a rectangular distribution, in accordance with equation (7).

$$u_{\mathrm{load}} = \frac{\bar{x}_{\mathrm{A0}} - \bar{x}_{\mathrm{L}}}{\sqrt{3}}. \tag{7}$$

The resulting uncertainty values at the calibration temperatures 5 °C, 29.7 °C, and 80 °C were reported in Table 5, Table 6, and Table 7, respectively.

3.12. The combined standard uncertainty of the bath characterization, ucchar

The characterization of the liquid bath was evaluated by estimating its combined standard uncertainty, u_{char}, which incorporates four main contributions: the standard uncertainty of the reference thermometer, the radial and axial temperature uniformity, the bath instability, and the loading effect. These contributions were combined according to equation (8), where the sensitivity coefficients c_1, c_2, c_3, and c_4 were all taken as unity, since each uncertainty component was already expressed in mK [11], [15]:

$$u_{\mathrm{char}}^2 = (c_1 \cdot u_{\mathrm{stand}})^2 + (c_2 \cdot u_{\mathrm{uniform}})^2 + (c_3 \cdot u_{\mathrm{instab}})^2 + (c_4 \cdot u_{\mathrm{load}})^2. \tag{8}$$

The calculated u_{char} were 2.70 mK at 5 °C, 3.17 mK at 29.7 °C, and 3.87 mK at 80 °C, as presented in Table 5, Table 6, and Table 7, respectively. These results clearly show that the liquid bath demonstrates excellent thermal stability and uniformity over the investigated temperature range. More importantly, the obtained u_{char} values remain well below the benchmark value of 10 mK, which is generally regarded as the upper limit for high precision calibration baths in temperature metrology.

3.13. Calibration of the PRT by comparison method

After characterization of the liquid bath, the PRT was calibrated by comparison against a calibrated 100-Ω SPRT. The bath temperature was set at each calibration point (5 °C, 29.7 °C, and 80 °C), and allowed to stabilize for one hour, then calibration was carried out for 10 minutes with readings taken every 30 s. The obtained 20 calibration results of the PRT were reported in Table 8. The mean, $\bar{x}$, and standard deviation, SD, were calculated and reported in the same table, as well [13].

3.14. Uncertainty of the PRT calibration

The temperature deviation, ΔT_{x}, indicated by the PRT from the temperature measured by the SPRT represents the calibration result. The mathematical model for this measurement is shown in equation (9) [13]:

$$\Delta T_{\mathrm{x}} = (T_{\mathrm{x}} - T_{\mathrm{s}}) + \delta T_{\mathrm{s}} + \delta T_{\mathrm{x}} + \delta T_{\mathrm{resol}} + \delta T_{\mathrm{EEW}} + \delta T_{\mathrm{wa}} + \delta T_{\mathrm{Hyst}}, \tag{9}$$

where

T_{x} - mean of 20 reading temperature of the thermometer under calibration;
T_{s} - temperature of the standard SPRT;
δT_{s} - uncertainty of the standard SPRT;
δT_{x} - standard uncertainty of the repeatability of the 20 readings;
$\delta T_{\mathrm{resol}}$ - correction due to the resolution of the thermometer under calibration;
δT_{EEW} - correction due to self-heating of the thermometer under calibration;
δT_{wa} - correction due to heat dissipation of thermometer under calibration;
δT_{Hyst} - correction due to hysteresis of thermometer under calibration.

The above-mentioned uncertainty sources in equation (9) are shown in the fishbone illustrated in Figure 4. Estimation of these uncertainty sources was carried out in accordance with the ISO GUM and the DKD-R 5-1 as described below [12], [13].

3.15. Uncertainty due to the standard SPRT

The uncertainty, δT_{s} due to the standard SPRT is a contribution from its calibration, drift, self heating, and calibration, and resolution of the super thermomer (indicator). The calibration uncertainty was divided by 2, while the drift and

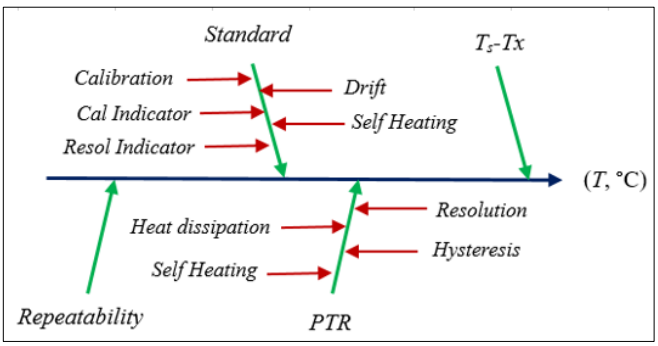


Figure 4. Fishbone showing the uncertainty sources of the PRT calibration.

Table 8. The calibration results of the PRT by comparison method at 5 °C, 29.7 °C, and 80 °C.

	5 °C			29.7 °C			80 °C		
	SPRT	**PTR**	**Error**	**SPRT**	**PTR**	**Error**	**SPRT**	**PTR**	**Error**
	5.07347	5.091	0.01753	29.39143	29.410	0.01857	79.89489	79.906	0.01111
	5.07352	5.090	0.01648	29.39148	29.408	0.01652	79.89485	79.904	0.00915
	5.07356	5.091	0.01744	29.39152	29.409	0.01748	79.89480	79.908	0.01320
	5.07359	5.094	0.02041	29.39155	29.411	0.01945	79.89475	79.907	0.01225
	5.07362	5.095	0.02138	29.39158	29.408	0.01642	79.89470	79.905	0.01030
	5.07365	5.091	0.01735	29.3916	29.410	0.01840	79.89464	79.908	0.01336
	5.07366	5.094	0.02034	29.39161	29.408	0.01639	79.89459	79.904	0.00941
	5.07366	5.095	0.02134	29.39161	29.406	0.01439	79.89454	79.908	0.01346
	5.07365	5.093	0.01935	29.39159	29.408	0.01641	79.89451	79.904	0.00949
	5.07363	5.093	0.01937	29.39159	29.407	0.01541	79.89449	79.908	0.01351
	5.07362	5.094	0.02038	29.39158	29.410	0.01842	79.89450	79.906	0.01150
	5.07365	5.091	0.01735	29.3916	29.408	0.01640	79.89470	79.908	0.01330
	5.07366	5.092	0.01834	29.39161	29.408	0.01639	79.89459	79.906	0.01141
	5.07362	5.094	0.02038	29.39159	29.409	0.01741	79.89454	79.905	0.01046
	5.07363	5.093	0.01937	29.39159	29.410	0.01841	79.89454	79.908	0.01346
	5.07359	5.091	0.01741	29.39162	29.411	0.01938	79.89469	79.906	0.01131
	5.07363	5.095	0.02137	29.39161	29.409	0.01739	79.89460	79.907	0.01240
	5.07364	5.092	0.01836	29.39160	29.408	0.01640	79.89459	79.906	0.01141
	5.07364	5.094	0.02036	29.39159	29.410	0.01841	79.89454	79.905	0.01046
	5.07363	5.093	0.01937	29.39158	29.409	0.01742	79.89461	79.907	0.01239
$\bar{x}$	5.07362	5.093	0.01918	29.39158	29.409	0.01727	79.89463	79.906	0.01167
SD	0.000049	0.0016	0.0016	0.000048	0.0013	0.00132	0.00012	0.0015	0.0015
$u_{\text{rept,°C}}$		0.00035			0.00029			0.00033	
$u_{\text{rept,mK}}$		0.35			0.29			0.33	

self-heating uncertainties were divided by $\sqrt{3}$ assuming a rectangular distribution. The uncertainty arising from the super thermometer (indicator) is from its calibration and resolution. The uncertainty of the calibration was multiplied by a sensitivity coefficient of 0.385 Ω to express it in a temperature unit [11]. In addition, the uncertainty due to the resolution was calculated using equation (2). The five contributions were combined using equation (3) giving a value of 2.052 mK, which was reported in Table 11, Table 12, and Table 13 for the calibration points 5 °C, 29.7 °C, and 80 °C, respectively.

3.16 Uncertainty of the repeatability, δT_x

The Type A uncertainty arising from the repeatability of the calibration results was calculated by dividing the standard deviation of the mean of the 20 temperature readings in Table 4 by the $\sqrt{n}$, according to equation (10)

$$u_{\text{rept}} = \frac{SD}{\sqrt{n}}. \tag{10}$$

The resulting values were found to be 0.35 mK, 0.29 mK, and 0.33 mK for the calibration points 5 °C, 29.7 °C, and 80 °C, respectively.

Table 9. The PRT calibration results for estimation of the heat dissipation uncertainty at 5 °C, 29.7 °C, and 80 °C.

	5 °C			29.7 °C			80 °C		
SPRT	**PTR 12.7 mm up**		**Error**	**SPRT**	**PTR 12.7 mm up**	**Error**	**SPRT**	**PTR 12.7 mm up**	**Error**
5.07461	5.096		0.02139	29.39142	29.408	0.016580	79.89456	79.903	0.00844
5.07460	5.095		0.02040	29.39144	29.407	0.015560	79.89457	79.905	0.01043
5.07458	5.092		0.01742	29.39146	29.403	0.011540	79.89458	79.907	0.01242
5.07455	5.091		0.01645	29.39149	29.408	0.016510	79.89459	79.904	0.00941
5.07451	5.093		0.01849	29.39153	29.410	0.018470	79.89460	79.907	0.01240
5.07432	5.096		0.02168	29.39156	29.409	0.017440	79.89460	79.905	0.01040
5.07426	5.092		0.01774	29.39159	29.410	0.018410	79.89460	79.906	0.01140
5.07419	5.091		0.01681	29.39162	29.408	0.016380	79.89459	79.903	0.00841
5.07413	5.095		0.02087	29.39164	29.407	0.015360	79.89458	79.908	0.01342
5.07407	5.093		0.01893	29.39165	29.410	0.018350	79.89457	79.905	0.01043
Ave (x_1)		mK	0.01902			0.01646			0.01072
Ave at 0-cm depth (x_2)		mK	0.01910			0.01694			0.01152
Heat dissipation		°C	0.00008			0.00048			0.00081
($x_2 - x_1$)		mK	0.081			0.484			0.808
$u_{\delta Twa}$		mK	0.047			0.28			0.47

3.17 Uncertainty due to the PRT resolution, δT_{Res}

The resolution of the readout of the PRT under calibration is 0.001 °C. This value was divided by 2 $\sqrt{3}$ according to equation (2) to calculate the uncertainty, assuming a rectangular distribution. The calculated value is then 0.29 mK.

3.18 Uncertainty due to the PRT self-heating, δT_{EEw}

When a platinum resistance thermometer (PRT) is measured, a small electrical current flows through the sensing element to determine its resistance, which generates a slight amount of heat inside the platinum wire and raises its temperature above that of the surrounding medium. As a result, the thermometer reading becomes slightly higher than the true temperature, an effect known as the self-heating error. Since the PRT under calibration was a digital one, it did not offer the possibility to measure the sensor resistance with different measuring currents. Therefore, the DKD-R 5-1, assumed uncertainty of 30 mK was considered with a rectangular distribution [13]. The associated standard uncertainty was then found to be 17 mK.

3.19 Uncertainty due to the heat dissipation, δT_{wa}

The heat dissipation error may occur during the calibration, because part of the thermometer dissipates heat to the external environment or to the equalizing block, and thus the entire sensor is not in perfect thermal equilibrium. Therefore, it is necessary to estimate the magnitude of this error and include it in the uncertainty budget. The SPRT and the PRT were submerged at the 0-cm depth, and after thermal equilibrium was reached, 10 temperature readings were taken over 5 minutes at 30-second intervals. The errors ($T_{\mathrm{PRT}} - T_{\mathrm{SPRT}}$) were calculated and averaged. After that, the PRT was pulled out by twice its diameter ($2 \cdot 6.35$ mm = 12.7 mm), and, after thermal equilibrium, 10 readings were taken at 30-second intervals [13]. The error was also calculated. The difference ($x_2 - x_1$) between the average temperature at the initial depth, 0 cm, and at pulling out 12.7 mm was calculated and taken as the heat dissipation uncertainty assuming a rectangular distribution. This procedure was repeated at the calibration points 5 °C, 30 °C, and 80 °C. The results obtained are shown in Table 9, and the associated standard uncertainties were then 0.047 mK, 0.28 mK, and 0.47 mK at 5 °C, 29.7 °C, and 80 °C, respectively.

3.20 Uncertainty due to hysteresis, δT_{Hyst}

Hysteresis in a platinum resistance thermometer (PRT) describes the variation in its indication at the same temperature, depending on whether that temperature is reached while heating up or while cooling down. This effect arises from certain physical characteristics of the probe, such as thermal expansion or delayed response. To evaluate hysteresis, the PRT was calibrated at the midpoint temperature of 37.5 °C within the working range of 5 °C to 80 °C, both rising and falling temperature sequences. The procedure began with setting the bath temperature to 5 °C (T_1), then raising it to the midpoint 37.5 °C (T_2), and to the maximum of 80 °C (T_3). Once the reading at 80 °C was completed, the bath was re-set back down directly to the midpoint, 37.5 °C (T_4), where 10 readings were recorded at 30 s at each set temperature. The measurements obtained at the midpoint during heating (T_2) and during cooling (T_4) are presented in Table 10. According to the DKD-R 5-1, the correction due to hysteresis of thermometer under calibration can be calculated using equation (11)

$$\delta T_{\mathrm{Hyst}} = \frac{T_4 - T_2}{\sqrt{3}}. \tag{11}$$

Table 10. The results of the PRT hysteresis at the middle temperature, 37.5 °C.

37.5 °C (T_2)			37.5 °C (T_4)		
SPRT	PTR	Error	SPRT	PTR	Error
37.15338	37.168	0.01462	37.15203	37.168	0.01597
37.15331	37.167	0.01369	37.15206	37.166	0.01394
37.15331	37.168	0.01469	37.15208	37.169	0.01692
37.15332	37.169	0.01568	37.15209	37.167	0.01491
37.15333	37.165	0.01167	37.15210	37.166	0.01390
37.15333	37.168	0.01467	37.15211	37.169	0.01689
37.15333	37.170	0.01667	37.15212	37.168	0.01588
37.15334	37.168	0.01466	37.15212	37.170	0.01788
37.15335	37.169	0.01565	37.15212	37.167	0.01488
37.15336	37.166	0.01264	37.15213	37.166	0.01387
	T_2	0.01446		T_4	0.01550
δT_{Hyst} (T_4–T_2) =1.040 mK					

Thus, the uncertainty due to hysteresis was found to be 1.040 mK [13]

3.21 The combined standard uncertainty of the PRT calibration, u_{cal}

The combined standard uncertainty, u_{cal} for the thermometer calibration by comparison against a standard SPRT was calculated at the calibration points 5, 29.7, and 80 °C according to equation (12), which includes the explicit contributions shown in the model equation (9). The c_1 to c_7 are the sensitivity coefficients and each of them equals 1 since the uncertainties are all expressed in mK [15]. The results of the uncertainty budget calculations are recorded in Table 11, Table 12, and Table 13. From these tables, it is clear that the u_{cal} values are: 26.00, 24.65, and 21.11 mK at 5, 29.7, and 30 °C, respectively.

$$\begin{aligned} u_{\mathrm{cal}}^2 = {} & (c_1 \cdot T_{\mathrm{x}} - T_{\mathrm{s}})^2 + (c_2 \cdot uT_{\mathrm{s}})^2 + (c_3 \cdot uT_{\mathrm{x}})^2 \\ & + (c_4 \cdot uT_{\mathrm{resol}})^2 + (c_5 \cdot uT_{\mathrm{EEW}})^2 + (c_6 \cdot uT_{\mathrm{wa}})^2 \\ & + (c_7 \cdot uT_{\mathrm{Hyst}})^2 . \end{aligned} \tag{12}$$

3.22 The overall combined uncertainty of the thermometer calibration, u_{c}

The overall combined standard uncertainty, u_{c} of the PRT calibration was obtained by combining the uncertainty contributions from the bath characterization u_{char} and the PRT calibration u_{cal} at the calibration points of 5 °C, 29.7 °C, and 80 °C. This combination was performed according to equation (13)

$$u_{\mathrm{c}} = \sqrt{(c_1 \cdot uT_{\mathrm{char}})^2 + (c_2 \cdot uT_{\mathrm{cal}})^2}\,, \tag{13}$$

using the values provided in Table 14, resulting in combined standard uncertainties of 26.14 mK, 24.85 mK, and 21.46 mK, respectively.

The expanded uncertainty, U_{exp} was calculated by multiplying each u_{c} by a coverage factor $k = 2$ corresponding to a level of confidence of approximately 95 %. After conversion of the expanded uncertainty from mK to °C, the PRT calibration results were reported as: 5 ± 0.052 °C, 29.70 ± 0.050, and 80 ± 0.043 °C.

These relatively small uncertainty values demonstrate that the calibration of the PRT was achieved with high accuracy. In particular, having expanded uncertainties below 0.06 °C across the investigated temperature range indicates that the measurement system and procedure are capable of delivering reliable and precise calibration measurement results.

4. CONCLUSIONS

The characterization of the liquid bath demonstrated excellent performance across all evaluated parameters, confirming its suitability for precision thermometer calibration. The long-term drift assessment at 20 °C showed remarkable stability within ± 1 mk over 12 hours, while short-term stability at 5 °C, 29.7 °C, and 80 °C was found to be 0.135 mK, 0.322 mK, and 0.779 mK, respectively. Radial non-uniformity values were limited to 0.48 mK, 0.69 mK, and 2.7 mK, while axial variations remained below 0.9 mK at all calibration points. The loading effect contributed only 0.26 mK, 0.45 mK, and 2.6 mK at the three temperatures, further confirming the bath stability under practical calibration conditions. Based on a full ISO GUM uncertainty evaluation, the combined standard uncertainties of the bath characterization were 2.70 mK, 3.17 mK, and 3.87 mK at 5 °C, 29.7 °C, and 80 °C, respectively. These values are well below the 10 mK threshold required for high-accuracy calibration baths. When applied to the calibration of a platinum resistance thermometer by comparison with a standard SPRT, the system achieved expanded uncertainties of ± 0.052 °C, ± 0.050 °C, and ± 0.043 °C at the same calibration points. These

Table 11. The calibration results of PRT by comparison with SPRT at 5 °C.

Source	Description	Estimate	u_{xi}	Unit	Distribution	Divisor	c_i	$c_i \cdot u_{xi}$
Standard SPRT	Cal SPRT	0	1.35	mK	Normal	2	1	1.1000
	Drift SPRT	0	2.37	mK	Rectangular	$\sqrt{3}$	1	1.7321
	Cal indicator	0	0.000050	Ω	Normal	2	0.385	0.019
	Resol indicator	0	0.0029	mK	Rectangular	$\sqrt{3}$	1	0.00289
	Self H. SPRT	0	0.45900	mK	Rectangular	$\sqrt{3}$	1	0.02194
							u_{stand}	2.763
Under calibration (PRT)	T_x-T_s		19.18400	mK	Normal	1	1	19.184
	Repeatability (T_x)	0	0.35	mK	Normal	1	1	0.35
	Resolution (δT_{resol})	0	0.29	mK	Rectangular	$\sqrt{3}$	1	0.29
	Self heating (δT_{EEW})	0	17.32	mK	Rectangular	$\sqrt{3}$	1	17.32
	Heat dissipation (δT_{wa})	0	0.047	mK	Rectangular	$\sqrt{3}$	1	0.047
	Hysteresis (δT_{Hyst})	0	0.60	mK	Rectangular	$\sqrt{3}$	1	0.60
	Combined standard uncertainty, $u_c(T_{cal})$						u_c	26.00

Table 12. The calibration results of PRT by comparison with SPRT at 29.7 °C.

Source	Description	Estimate	u_{xi}	Unit	Distribution	Divisor	c_i	$c_i \cdot u_{xi}$
Standard SPRT	Cal SPRT	0	1.65	mK	Normal	2	1	1.1000
	Drift SPRT	0	2.37	mK	Rectangular	$\sqrt{3}$	1	1.7321
	Cal indicator	0	0.000050	Ω	Normal	2	0.385	0.019
	Resol indicator	0	0.0029	mK	Rectangular	$\sqrt{3}$	1	0.00289
	Self H. SPRT	0	0.45900	mK	Rectangular	$\sqrt{3}$	1	0.02194
							u_{stand}	2.92
Under calibration (PRT)	T_x-T_s		17.27350	mK	Normal	1	1	17.2735
	Repeatability (T_x)	0	0.29	mK	Normal	1	1	0.29
	Resolution (δT_{resol})	0	0.29	mK	Rectangular	$\sqrt{3}$	1	0.29
	Self heating (δT_{EEW})	0	17.32	mK	Rectangular	$\sqrt{3}$	1	17.32
	Heat dissipation (δT_{wa})	0	0.28	mK	Rectangular	$\sqrt{3}$	1	0.28
	Hysteresis (δT_{Hyst})	0	0.60	mK	Rectangular	$\sqrt{3}$	1	0.60
	Combined standard uncertainty, $u_c(T_{cal})$						u_c	24.65

Table 13. The calibration results of PRT by comparison with SPRT at 80 °C.

Source	Description	Estimate	u_{xi}	Unit	Distribution	Divisor	c_i	$c_i \cdot u_{xi}$
Standard SPRT	Cal SPRT	0	1.65	mK	Normal	2	1	1.1000
	Drift SPRT	0	2.37	mK	Rectangular	$\sqrt{3}$	1	1.7321
	Cal indicator	0	0.000050	Ω	Normal	2	0.385	0.019
	Resol indicator	0	0.0029	mK	Rectangular	$\sqrt{3}$	1	0.00289
	Self H. SPRT	0	0.45900	mK	Rectangular	$\sqrt{3}$	1	0.02194
							u_{stand}	2.92
Under calibration (PRT)	T_x-T_s		11.66700	mK	Normal	1	1	19.184
	Repeatability (T_x)	0	0.33	mK	Normal	1	1	0.35
	Resolution (δT_{resol})	0	0.29	mK	Rectangular	$\sqrt{3}$	1	0.29
	Self heating (δT_{EEW})	0	17.32	mK	Rectangular	$\sqrt{3}$	1	17.32
	Heat dissipation (δT_{wa})	0	0.47	mK	Rectangular	$\sqrt{3}$	1	0.047
	Hysteresis (δT_{Hyst})	0	0.60	mK	Rectangular	$\sqrt{3}$	1	0.60
	Combined standard uncertainty, $u_c(T_{cal})$						u_c	21.11

Table 14. The overall PRT calibration uncertainty.

Calibration points (°C)		5	29.7	80
Bath characterization uncertainty, u_{char} ()	mK	2.70	3.14	3.87
Thermometer calibration uncertainty, u_{cal} ()	mK	26.00	24.65	21.11
The combined standard uncertainty, u_c	mK	26.14	24.85	21.46
U_{exp}	mK	52.28	49.70	42.92
	°C	0.052	0.050	0.043
	%	1.046	0.17	0.054

results clearly demonstrate that the studied liquid bath provides a highly stable, uniform and reliable thermal environment, ensuring accurate thermometer calibrations with small uncertainties. The reported liquid bath characterization and thermometer calibration with their detailed uncertainty estimations will be of considerable value to the temperature calibration laboratories.

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