THE NEW GRAVITY ZONE CONCEPT IN EUROPE FOR WEIGHING INSTRUMENTS UNDER LEGAL CONTROL

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Abstract
A new European gravity zone concept has been developed by WELMEC, the European Cooperation in Legal Metrology. This concept is independent of political or administrative borders and is accepted by all WELMEC member states regardless of existing national regulations concerning gravity zones. The new concept is based on gravity zones defined in terms of the geographical latitude φ and the height above sea level h, on a standardised gravity formula used for the adjustment of a weighing instrument to a reference gravity value, and on a uniform criterion for calculating the maximum permissible variation $\Delta g/g$ within an individual gravity zone. The concept is of advantage especially for weighing instruments of class III. It offers an option that can, but must not be chosen by a manufacturer.

1 INFLUENCE OF GRAVITY ON PRECISION WEIGHING

The local gravity value plays an important role in precision measurements of many mechanical quantities. Especially modern electronic weighing instruments for mass determinations with relative uncertainties in the range of 10^{-3} to 10^{-7}, or even less, are sensitive to gravity variations depending on the geographical latitude φ and the height above sea level h. Both for high-precision applications (relative uncertainties less than 10^{-4}) and for applications of medium accuracy (relative uncertainties 10^{-3} to 10^{-4}) the respective instruments must be adjusted at their exact place of use. This is no problem for high-precision balances which normally use electromagnetically compensated load cells and which are adjusted either by an incorporated adjustment weight or by an external, calibrated precision weight.

Weighing instruments of medium accuracy, however, which normally use strain-gauge load cells, are used in large numbers of pieces in industry and trade for commercial transactions, ie. for applications under legal control. The problem here is that from the commercial point of view the use of an incorporated adjustment weight is too expensive and from the legal point of view the adjustment by an external weight at the place of use is normally not allowed for the user, but only for an authorised person. Thus, European regulations foresee a verification in two stages: the first one comprising all examinations that are gravity-independent, which can be carried out at the manufacturers works, and a second one, comprising essentially the final adjustment by an authorised person at the place of use. Only in case that gravity zones are defined in the respective country and if the weighing instrument is marked with a gravity zone the user is allowed to freely move his instrument in this particular zone without making the verification invalid.

2 NATIONAL REGULATIONS FOR GRAVITY ZONES

There are only a few countries in Europe, eg. Austria, Germany [1] and Italy [2], that have national regulations which define gravity zones depending on the accuracy of the weighing instrument used. These enable a manufacturer to correctly adjust an instrument by simply using the legally fixed mean gravity values for one, two or several zones.

The definition of sufficiently large gravity zones is possible because the global variation of gravity is relatively small: At constant height h the maximum variation $\Delta g_{\text{pole-equator}}$ is about

0,05 ms^{-2} or 0,5%, respectively. At constant geographical latitude φ the gravity value decreases by about 0,003 ms^{-2} per 1000 m increase in h. The definition of gravity zones is therefore important mainly for countries with a large north-south extent or with a rough topography. In addition, the size and number of gravity zones depend on the fraction of the maximum permissible error that an instrument is granted when it is moved within a zone. In Germany and Italy the following criterion is applied:

$$\Delta g \,/\, g \;\leq\; mpe \,/\, (n \cdot e), \qquad (1)$$

where $\Delta g / g$ is the allowed relative variation of g, mpe is the maximum permissible error of the weighing instrument in question, and n is its number of verification scale intervals e (n = maximum load Max divided by the verification scale interval e, which is normally equal to the digital resolution d).

Table 1 shows - both for Germany and Italy - the maximum number n depending on the accuracy class (II or III) and depending on whether the instrument is adjusted for use in the entire country or in two adjacent zones or in one individual zone. The table also shows that the allowed relative variations $\Delta g / g$ are very similar in both countries.

Table 1: *Allowed relative variations $\Delta g/g$ and maximum numbers of verification scale intervals n for weighing instruments of accuracy classes II and III according to the national regulations in Germany and Italy [2].*

Germany		*n*	
	$\Delta g/g$	class II	class III
Country	±0,00050	≤ 1000	≤ 3000
Adjacent zones	±0,00034	≤ 2000	≤ 5000
Individual zone	±0,00021	≤ 3300	≤ 10000

Italy		*n*	
	$\Delta g/g$	class II	class III
Country	±0,00051	≤ 1000	≤ 3000
Adjacent zones	±0,00036	≤ 2000	≤ 5000
Individual zone	±0,00022	≤ 3000	≤ 8000

National regulations for gravity zones are normally defined in terms of administrative boundaries rather than purely geographical or physical aspects. Fig. 1 shows the actual gravity zones for Germany and Italy as an example.

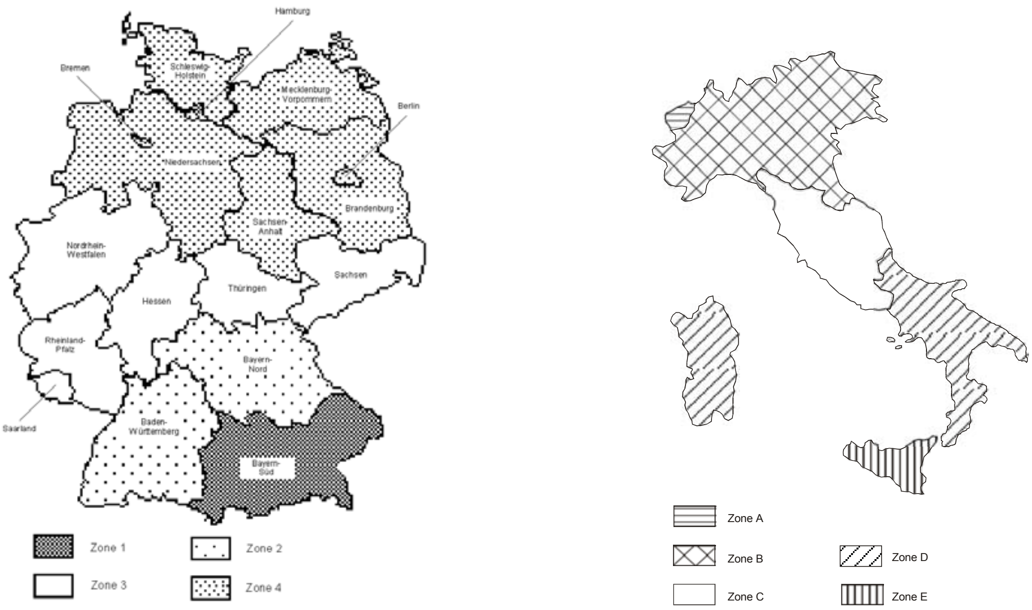

Figure 1: *Gravity zones in Germany and Italy based on administrative boundaries*

3 THE NEW EUROPEAN GRAVITY ZONE CONCEPT

In the year 2000 a new European gravity zone concept has been developed and published by WELMEC [3]. This concept is applicable both in countries with and without national regulations concerning gravity zones. It is independent of political or administrative borders and is based on

- the definition of an individual gravity zone for a weighing instrument, only depending on the relevant physical influence factors φ and h, and the accuracy of the instrument,
- a standardised gravity formula which is used for calculating the reference gravity value g_R to which the instrument is adjusted,
- a uniform criterion for calculating the maximum permissible variation $\Delta g/g$ within the zone.

It is worth mentioning that the new concept offers an option that can, but must not, be chosen by a manufacturer. It has the advantage that it is accepted by all WELMEC member states regardless of existing national regulations.

A gravity zone - which contains the intended place of use of the weighing instrument - is defined as the strip that is limited by a north and a south degree of latitude, φ_1 and φ_2, and by an upper and lower limit of the height above sea level, h_1 and h_2. In order to keep clarity, the values for φ shall be chosen as multiples of 1° (exceptionally 0,5° is also allowed) and the values for h as multiples of 100 m.

The reference gravity value g_R of this zone is then calculated in terms of the geographical latitude φ and the height above sea level h according to the formula [3,4]:

$$g = 9{,}780\,318\,(1 + 0{,}005\,3024 \sin^2\varphi - 0{,}000\,0058 \sin^2 2\varphi) - 0{,}000\,003085 \cdot h \quad \mathrm{m\,s^{-2}} \qquad (2)$$

Eq. (2) originates from a recommendation of the International Association of Geodesy (IAG) of 1967 [5], and is part of the so-called normal gravity field which is the best possible approximation of the earth's gravitional field. The uncertainties and expected deviations of this approximation with regard to exact gravity measurements will be discussed in Chapter 4.

The maximum permissible variation $\Delta g/g$ is calculated using the criterion:

$$n\,(\Delta g_\varphi + \Delta g_h) / g_R \leq mpe / (3e) \qquad (3)$$

When comparing eq. (3) with eq. (1) it can easily be seen that the criterion used in the WELMEC concept is more stringent than existing national criteria by a factor of 3. This ensures full compatibility with the existing national regulations and was the prerequisite for the general acceptance of the new concept by all WELMEC member states.

The complete procedure of defining a gravity zone for a given weighing instrument is devided into the following steps:

(A) Definition of appropriate zone boundaries:

φ_1, φ_2	as multiples of 1° (0,5°)	latitudes of zone boundaries
h_1, h_2	as multiples of 100 m	heights above sea level

(B) Computation of the maximum gravity variation in the defined zone:

$h_m = \frac{1}{2}(h_1 + h_2)$	mean value of height h	(4)
$\varphi_m = \frac{1}{2}(\varphi_1 + \varphi_2)$	mean value of latitude φ	(5)
$g_R = g(\varphi_m, h_m)$	reference value of gravity in the zone	(6)
$\Delta g_\varphi = \frac{1}{2}\lvert g(\varphi_1, h_m) - g(\varphi_2, h_m)\rvert$	max. variation due to a change in φ	(7)
$\Delta g_h = \frac{1}{2}\lvert g(\varphi_m, h_1) - g(\varphi_m, h_2)\rvert$	max. variation due to a change in h	(8)

(C) Check using eq. (3) whether the maximum permissible variation ($\Delta g_\varphi + \Delta g_h$) / g_R is kept.

Because of eqs. (3), (7) and (8) a broader strip (φ_1 - φ_2) can be compensated by narrower limits for h, and vice versa. The key factor for the maximum possible size of a zone is, however, the *mpe* of the weighing instrument itself. Table 2 shows possible gravity zones that include the Paris area as an example, for three different types of weighing instruments. In addition, Fig. 2 shows an example of the gravity zone (strip) with φ_1 = 48° and φ_2 = 50°.

Table 2: *Examples of gravity zones that include the Paris area (France), for three different types of weighing instruments*

Weighing instrument	**Class III**		**Class II**
divisions ***mpe***	***n* = 1000** **1,0 *e***	***n* = 3000** **1,5 *e***	***n* = 10.000** **1,0 *e***
φ_1 [°]	47	48	49
φ_2 [°]	51	50	49,5
h_1 [m]	0	0	0
h_2 [m]	800	400	100
$g(\varphi_1, h_m)$ [m s^{-2}]	9,806766	9,808285	9,809646
$g(\varphi_2, h_m)$ [m s^{-2}]	9,810350	9,810078	9,810094
$g(\varphi_m, h_1)$ [m s^{-2}]	9,809801	9,809801	9,810025
$g(\varphi_m, h_2)$ [m s^{-2}]	9,807333	9,808567	9,809716
g_R [m s^{-2}]	9,808567	9,809184	9,809870
$n\,(\Delta g_\varphi + \Delta g_h) / g_R \leq mpe / (3e)$	0,31 ≤ 0,33	0,46 ≤ 0,50	0,31 ≤ 0,33

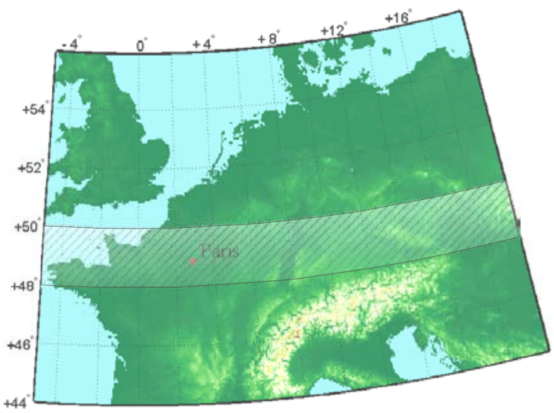

Figure 2: *Example of one gravity zone (strip) with* φ_1= 48° *and* φ_2= 50° *that includes the Paris area (France)*

The example demonstrates that with the WELMEC concept it is not necessary to know exactly the g value of the intended place of use (here Paris); it would be sufficient if the three weighing instruments were adjusted to the respective calculated reference value g_R. The corresponding gravity zone would then be marked, for instance, on the dataplate of the instrument either in the form 47-51:0-800, 48-50:0-400 or 49-49,5:0-100, or equivalently in the form 47-51≡0-800, 48-50≡0-400 or 49-49,5≡0-100.

The example also demonstrates that the WELMEC concept is practically restricted to class III instruments, because for class II the strong criterion, eq. (3), limits the number of divisions to a maximum of n = 10.000 for the smallest possible zone ($\Delta\varphi$ = 0,5°, Δh = 100 m).

4 CONSIDERATION OF UNCERTAINTIES AND ANOMALIES

The major source of uncertainty of the new gravity zone concept is the standardised gravity formula, eq. (2). It approximates the earth's gravitional field which is influenced by both the latitude and the height. These two factors basically split the linear formula into two parts. The first component only depends on the latitude φ and is constructed as a simplified series expansion. The IAG recommends this series expansion for applications with relative accuracy requirements up to $5 \cdot 10^{-7}$. Series expansions of higher order allow relative uncertainties of even $1 \cdot 10^{-9}$ to be reached. In any case can the uncertainty contribution from the first component be neglected here.

The second part is the more critical one, because it contains not only the height dependency - which can be linearly approximated - but also the disturbances due to local topographic influences. There are sophisticated models that are able to take into account local and regional topographic influences [5]; for medium accuracy applications like the WELMEC gravity concept, however, these are too complex and practically not applicable. For the uncertainty analysis of the second part of the gravity formula, eq. (2), a sample of almost 60.000 established measurement data taken from a European database [6] kept at the Institute of Geodesy (IfE, Hannover, Germany) has been analysed, see Fig. 3. Here the anomalies Δg (= differences between the g values calculated with the normal gravity formula and the g values taken from the database) are plotted against the corresponding heights above sea level. A regression analysis yields the linear gravity gradient

$$\mathrm{d}g \,/\, \mathrm{d}h = -3{,}082 \cdot 10^{-6}\ \mathrm{m \cdot s^{-2} \cdot m^{-1}} \qquad (10)$$

which is in very good agreement with the free-air gradient of the gravity formula, eq. (2), that is used for the WELMEC concept.

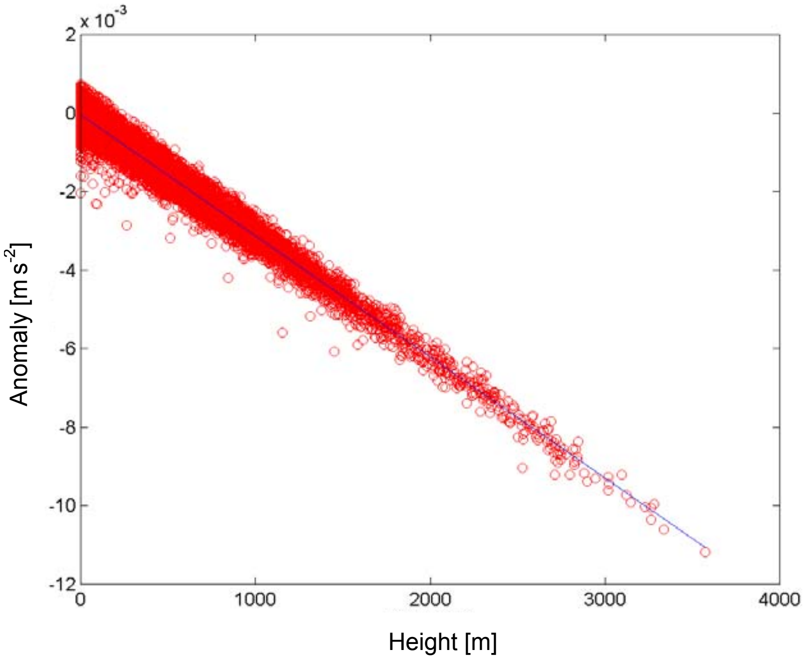

Figure 3:

Regression analysis of the anomalies Δg as a function of the height above sea level h for a sample of almost 60.000 gravity data for Western Europe

It can easily be recognised that the scatter of Δg decreases with increasing h, which can be explained by the reduced influence of local topographic effects with increasing distance from the earth's surface, and that the anomalies reach maximum values of not more than about $2 \cdot 10^{-3}$ m·s^{-2} or $2 \cdot 10^{-4}$ relatively. An analysis of the 60.000 data reveals that 87,5% of the anomalies are smaller than $0{,}5 \cdot 10^{-4}$ (relatively), 97,4% are smaller than $1{,}0 \cdot 10^{-4}$ and 99,5% are smaller than $1{,}5 \cdot 10^{-4}$. Only 0,5% of the anomalies are larger than $1{,}5 \cdot 10^{-4}$ (relatively); these are exclusively related to extreme locations in the Alps.

Another analysis was performed using a much smaller sample of 46 selected Western European cities, see Fig. 4. These cities are considered as a representative sample of locations where weighing instruments, requiring a gravity adjustment, are used for legal applications. Table 3 contains the complete list of locations, including their geographical positions, the g_W values ('WELMEC') calculated with the gravity formula, eq. (2), the g_M values ('Database') taken from the European database [6] and the relative difference between both.

As a result it could be proved that except for 3 cities in Italy (Bologna, Milano and Catania) all relative deviations are smaller than $5{\cdot}10^{-5}$ as it is considered sufficient for the WELMEC concept [3]. Even the largest relative deviation (Bologna) is still in the order of $1{\cdot}10^{-4}$. In order to keep the relative deviations generally below $5{\cdot}10^{-5}$ it is recommended that for these critical regions either a small correction is applied to the calculated g_R value, or the zone is defined such that the respective g_M value of Table 3 can be taken as reference value g_R.

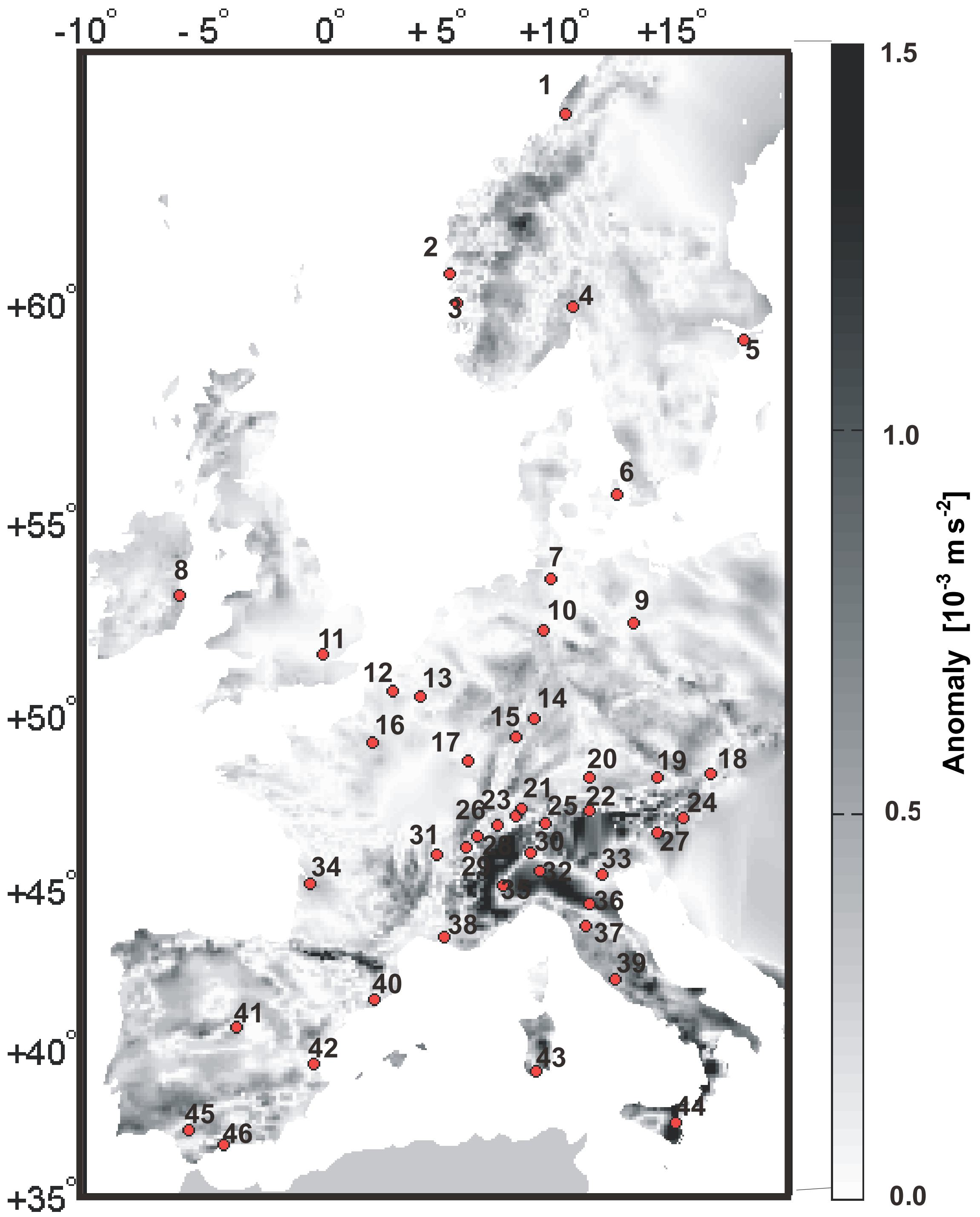

Figure 4: *Map of Western Europe showing the maximum anomalies Δg (black colour) in the region of the Alps and the North Italian lowlands. Also shown are the locations of 46 European cities, the numbering refers to Table 3.*

Table 3: *Selection of European cities that are considered as a representative sample of locations for weighing instruments used for trade purposes (legal applications).*

No	Location	Latitude	Longitude	Height	g_W by 'WELMEC'	g_M from 'Database'	$(g_M - g_W)/g_M$
		[°]	[°]	[m]	[ms^{-2}]	[ms^{-2}]	
1	Trondheim	63,78	10,33	135	9,821932	9,821611	0,000033
2	Bergen	60,58	5,47	60	9,819538	9,819447	0,000009
3	Stavanger	59,97	5,75	25	9,818777	9,819078	-0,000031
4	Oslo	59,89	10,69	29	9,819279	9,819001	0,000028
5	Stockholm	59,18	17,98	71	9,818175	9,818313	-0,000014
6	Copenhagen	55,64	12,58	2	9,815483	9,815614	-0,000013
7	Hamburg	53,55	9,70	10	9,813708	9,813801	-0,000010
8	Dublin	53,11	-6,04	15	9,813695	9,813400	0,000030
9	Berlin	52,42	13,25	34	9,812683	9,812741	-0,000006
10	Hannover	52,24	9,40	55	9,812749	9,812517	0,000024
11	London	51,59	0,04	19	9,812031	9,812059	-0,000003
12	Lille	50,62	3,02	22	9,811180	9,811188	-0,000001
13	Brussel	50,48	4,21	143	9,810768	9,810691	0,000008
14	Frankfurt	49,87	9,05	91	9,810476	9,810311	0,000017
15	Mannheim	49,34	8,28	95	9,809552	9,809821	-0,000027
16	Paris	49,20	2,17	35	9,809649	9,809881	-0,000024
17	Nancy	48,67	6,22	212	9,808824	9,808859	-0,000004
18	Wien	48,31	16,57	170	9,808540	9,808667	-0,000013
19	Linz	48,23	14,25	254	9,808110	9,808334	-0,000023
20	München	48,21	11,39	520	9,807355	9,807500	-0,000015
21	Zürich	47,35	8,50	408	9,806695	9,807066	-0,000038
22	Innsbruck	47,27	11,37	574	9,806356	9,806482	-0,000013
23	Luzern	47,13	8,23	436	9,806360	9,806781	-0,000043
24	Graz	47,05	15,41	365	9,807132	9,806928	0,000021
25	Chur	46,93	9,53	587	9,805680	9,806135	-0,000046
26	Bern	46,87	7,47	540	9,805937	9,806226	-0,000029
27	Klagenfurt	46,62	14,25	446	9,806306	9,806290	0,000002
28	Lausanne	46,52	6,57	495	9,805838	9,806048	-0,000021
29	Genf	46,20	6,12	375	9,805819	9,806129	-0,000032
30	Lugano	46,02	8,85	272	9,806439	9,806284	0,000016
31	Lyon	45,96	4,90	169	9,806578	9,806550	0,000003
32	Milano	45,48	9,27	127	9,805502	9,806247	-0,000076
33	Padua	45,40	11,89	13	9,806444	9,806522	-0,000008
34	Bordeaux	45,12	-0,53	5	9,805996	9,806293	-0,000030
35	Torino	45,08	7,68	239	9,805401	9,805535	-0,000014
36	Bologna	44,50	11,36	55	9,804490	9,805578	-0,000111
37	Firenze	43,80	11,24	51	9,805141	9,804957	0,000019
38	Marseille	43,46	5,20	10	9,804856	9,804777	0,000008
39	Roma	42,19	12,47	20	9,803963	9,803598	0,000037
40	Barcelona	41,53	2,25	20	9,803112	9,803006	0,000011
41	Madrid	40,65	-3,62	670	9,799890	9,800212	-0,000033
42	Valencia	39,47	-0,38	138	9,800810	9,800802	0,000001
43	Cagliari	39,22	9,12	26	9,801186	9,800926	0,000027
44	Catania	37,50	15,08	11	9,800349	9,799459	0,000091
45	Sevilla	37,21	-5,71	10	9,798774	9,799206	-0,000044
46	Malaga	36,72	-4,20	8	9,799508	9,798789	0,000073

5 CONCLUSION

The new gravity concept of WELMEC allows weighing instruments under legal control to be verified in one stage at the manufacturers works for any zone in Europe and independent of still existing national regulations. The exact place of use need not be known; the only condition to be observed by the user is that the place of use lies within the gravity zone that is marked on the instrument. The new concept offers an optional and flexible possibility for a manufacturer to mark his instruments with individually chosen gravity zones which are defined in terms of the geographical latitude φ and the height above sea level h.

By a regression analysis of almost 60.000 established measurement data taken from a European database it could be proved that the gravity formula used for the adjustment of weighing instruments under the new WELMEC concept is sufficiently accurate. In addition an analysis of the data of 46 selected Western European cities was performed, these cities being considered as a representative sample of locations of weighing instruments that require a gravity adjustment and are used for legal applications. It could be proved that except for 3 cities in Italy all relative deviations are smaller than $5 \cdot 10^{-5}$ as it is considered sufficient for the WELMEC concept. Even the largest relative deviation is still in the order of $1 \cdot 10^{-4}$ and can easily be taken into account.

Because of the rather strong criterion used for the calculation of the maximum permissible variation $\Delta g/g$ within one gravity zone the new concept is practically restricted to weighing instruments of class III.

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