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THE SETUP AND PERFORMANCE EVALUATION OF ACCELEROMETER COMPARISON CALIBRATION SYSTEM IN NML TAIWAN

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Abstract – Accelerometer comparison calibration technique involves the back-to-back coupling the accelerometer under test directly to a traceable reference standard accelerometer and both accelerometers will be experienced the same motion from shaker. Thus the sensitivity of standard accelerometer can be transferred to the test accelerometer via a calibration constant from the output signals. The results of the relative expand uncertainty for the voltage sensitivities for accelerometer are less than 1.4 %, 1.9 % and 2.9 % within frequency range from 50 Hz to 1.5 kHz, from 1.5 kHz to 5 kHz, and from 5 kHz to 7 kHz, respectively. This paper will present the construction, the performance evaluation of the calibration system in NML, CMS/ITRI.

Keywords: accelerometer calibration, ISO 16063-21

1. INTRODUCTION

Precision vibration measurements depend on accurate, repeatable and stable calibration method. Standardization of calibration test equipment and measurement techniques are ensuring more accurate and repeatable measurement. Vibration calibration is the art about the measurement the sensitivity of accelerometer. The uses of periodically oscillatory (sinusoidal) excitation provided by an electromechanical exciter or shaker and the measurement of the accelerometer sensitivity are described by the ISO 16053-21, Method for the calibration of vibration and shock transducers – Part 21: Vibration calibration by comparison to a reference transducer[1]. Accelerometer calibration using reference (or transfer) standard is the most common and efficiency method for most typical accelerometer users. The accelerometer under test is directly back-to-back mounted to a reference accelerometer and both experience the same sinusoidal motion driven by shaker under specific amplitude under certain frequency. The ratio of the output signals provide the calibration constant for the accelerometer under test, thus the sensitivity of the sensor under test (SUT) is calculated through the calibration constant and the sensitivity of the reference accelerometer[2]. This paper firstly presents the principle of accelerometers back-to-back calibration technique. Secondly, the components within the accelerometer sensitivity measuring system are introduced. Thirdly, the procedure and information for the performance evaluation of the calibration system in NML is illustrated.

2. HARDWARE DESCRIPTION

For accelerometer sensitivity comparison calibration, the sensor under test, mounted "back-to-back" to the secondary standard is excited sinusoidally, steeping through the desired frequencies at the desired acceleration amplitudes. The measurement components are divided into two distinct subsystems: excitation and measurement system as shown in Fig. 1 and Fig. 2. The excitation subsystem consists of electrodynamic shaker and associated amplifier is designed to produce the test conditions (constant vibration level at a specified amplitude and frequency). The measurement subsystem is composed of computer data acquisition and control, back-to-back reference standard, and signal conditioners for different type of accelerometers. This subsystem will measure the voltage output ratio of the two accelerometers and calculated the sensitivity of the sensor under test.

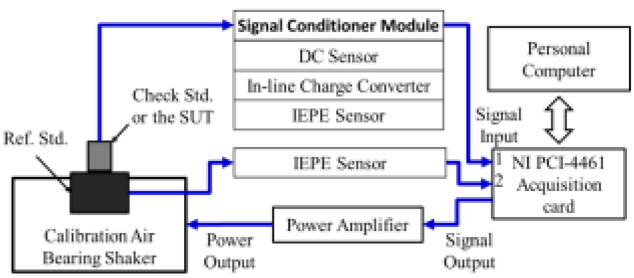

Fig. 1. Accelerometer calibration system module blocks

Table 1 Standards and/ or instrument of calibration instruments

Standards or instruments	Mfr.	Model	Serial Number
Checking Standard/ signal conditioner	PCB	353B04/442A102	LW173019/365
Data Acquisition card	NI	PCI-4461	1918FC4
Reference Standard/ Signal conditioner	PCB	080A200/442A102	170048/366
Vibration Exciter	PCB	396C10	772

Fig. 2 The calibration station configuration

3. SYSTEM EVALUATION

The main purpose of system evaluation is for the estimation and calculation of measurement uncertainty. Such uncertainty will express dispersion of the measurand. The description of measurement results must include the expression of uncertainty; otherwise such measurement process would not be a complete one. The uncertainty of a typical accelerometer calibration system falls into three primary categories: the voltage ratio measurement, the transfer standard accelerometer, and the exciter performace.

3.1. Measurement principle

There are many factors contributing to accelerometer calibration uncertainty. Consider the measurement equation for accelerometer calibration in Eq.(1)

$$S_u = \frac{V_u}{V_r} \times S_r = V_R \times S_r \quad (1)$$

where S_u is the voltage sensitivity of the accelerometer set to be calibrated (mV/(m/s^2)),

S_r is the voltage sensitivity of the reference standard accelerometer set (mV/(m/s^2)),

V_u is the output voltage of the accelerometer set to be calibrated measured (mV),

V_r is the output voltage of the working standard accelerometer set measured (mV),

V_R is ratio of V_u and V_r

Equation (1) states the ratio of output voltages of both reference standard accelerometer and SUT which must be equal to the ratio of their respective sensitivities. Because of the ratio nature of the voltage measurement, it is easy to see that if an external factor effects the voltage measurement.

3.2. Uncertainty analysis

We take the check standard accelerometer (PCB 353B04) to evaluate the vibration calibration system. If each input X_i of S_r, and V_R is uncorrelated, the variance of measurement result S_u can be calculated in the following way:

$$u_c^2(S_u) = \sum \left(\frac{\partial S_u}{\partial X_i}\right)^2 u^2(X_i) = \left(\frac{\partial S_u}{\partial S_r}\right)^2 u^2(S_r) + \left(\frac{\partial S_u}{\partial V_R}\right)^2 u^2(V_R) \quad (2)$$

In Eq(2), $u(S_r)$ and $u(V_R)$ is the standard uncertainty of the working standard accelerometer and the affection to output voltage ratio from the checking standard accelerometer and reference standard accelerometer. Each sensitivity coefficient related to the partial derivative with each input *Xi* is deduced as follows:

$$\frac{\partial S_u}{\partial S_r} = V_R, \quad \frac{\partial S_u}{\partial V_R} = S_r \quad (3)$$

3.3. Error Sources and uncertainty contributors

The error sources in the uncertainty estimation include the randoim error, shaker transverse and rocking motion, sensitivity uncertainty from the primary standard and from drift of the reference standard accelerometer, and the uncertainty in the voltage ratio measurement. The evaluation of measurement uncertainty in this document comprises Type A and Type B evaluations of uncertainty, which are defined as follows:

(A) Standard uncertainty is the uncertainty associated with the measurement result is expressed by using one standard deviation.

(B) Type A standard uncertainty (random error) is results in deviations from calibration to calibration from a series of measurements on a check standard accelerometer. Influences contribute to the random error inculuded: (i)cabling and connector, (ii)mounting torque variation, (iii)operator technique, (iv)temperature and humidty fluctuations, (v)electrical hum and noise in the measuremenr chain. in the statistical analysis by using one standard deviation.

(C) Type B standard uncertainty (systemic error) is obtained by an assumed probability density function based on the degree of belief that an event will occur and is estimated by scientific judgement coupled with all available information using non-statistical means. The transverse and rocking motion from the shaker $u(V_{R\_Rocking})$, the standard uncertainty from the reference standard accelerometer calibration report, and the voltage ratio error from the voltage measuring device (National Instruments , PCI-4461)

3.3.1 Type A relative standard uncertainty from the voltage ratio measurement

The type A standard uncertainty is combined of voltage ratio repetition $u_{A1}(V_R)$ and $u_{A2}(V_R)$ from long term and short term, respectively. This type A uncertainty is also called random error, and is determined by iterative test of a check standard on different days (almost 50 days in this case). Over frequency range 50 Hz to 7 kHz, 39 test (n_1=39) were performed on the check standard accelerometer were performed on the calibration system. Each calibration consisted of 19 sensitivities determined at 19 different exciting frequencies.

(i) Relative standard uncertainty from the long term repeatability, $u\ (V_{Rep\_1})$

The relative deviation error from the 39 repeatedly voltage ratio readings, V_{Rep_1}, is determined through following equation

$$V_{Rep\_1} = \frac{V_{SDV\_Rep\_1}}{\sqrt{n_1}} \times 100 \quad (4)$$

where $V_{SDV\_Rep\_1}$ is the standard deviation of voltage ratio in the n_1 repeated measnurements and n_1=39

Next ,the relative standard uncertainty of the long term repeatability $u(V_{Rep\_1})$ cound be estimated by (5). The measuring and caluating results of $V_{SDV\_Rep\_1}$ and $u(V_{Rep\_1})$ are listed in Table 2.

$$u(V_{Rep\_1}) = \frac{V_{Rep\_1}}{\sqrt{3}} \quad (5)$$

(ii) Relative standard uncertainty from the short term repeatability, $u\ (V_{Rep\_2})$

The short term repeatibility is detemined from 5 measuements. The uncertainty range of short term repetition, V_{Rep_2}, is determined following:

$$V_{Rep\_2} = \frac{V_{SDV\_Rep\_2}}{\sqrt{n_2}} \times 100 \quad (6)$$

In Equation (6) the $V_{SDV\_Rep\_2}$ is the standard deviation of voltage ratio in the short term calibration tests, and n_2 =5. Then the relative standard uncertainty of the short term repeatability $u(V_{Rep\_2})$ cound be estimated

$$u(V_{Rep\_2}) = \frac{V_{Rep\_2}}{\sqrt{3}} \quad (7)$$

Table 2 The uncertainties from repetitions measurement

Frequency Range	50 to 1.5k	1.5k to 5k	5k to 7k
V_{SDV_REP1}	0.0031	0.0111	0.0230
$u(V_{rep\_1})$	0.0286	0.1020	1.0890
$V_{SDV\_Rep\_2}$	0.0012	0.0007	0.0004
$u(V_{REP\_2})$	0.0294	0.0162	1.0890

3.3.2 Type B standard uncertainty in the voltage ratio measurement

The typed B uncertainty is estimated from every possible error contributions ubi(VR) from test standards or instruments referring to calibration reports or equipment specifications. The type B standard uncertainty will be the combination of all the indivial systemic contributors. In this system evaluation they are (i)the uncertainty from the traceable reference standard calibration, (ii) the uncertainty from the voltage ratio measuring device (iii) the uncertainty from the shaker transverse and rocking motion,

(i) Relative standard uncertainty from the reference standard traceability to primary standard

$$u(S_{Trace}) = S_r \times U_{Sen\_Trace} \quad (8)$$

where S_r is the voltage sensitivity calibration result
U_{Sen_Trace} is the relative expanded uncertainty
K is the coverage factor, K=2

The voltage sensitivity of the reference standard accelerometer, S_r, the relative expanded uncertainty $U_{sen,\text{trace}}$ and the cover factor, K, could refer to the calibration report[6].

Table 3 Uncertainties from reference standard calibration

Frequency [Hz]	50 to 1.5k	1.5k to 5k	5k to 7k
Relative Expand Uncertainty ,U [%]	1.0	1.5	2

(ii) Relative standard uncertainty of voltage ratio traceability

The calibration of voltage measurement accuracy is performed on periodic basis. Voltages with different traceable amplitutes and frequencies were acquired by voltage ratio acquisition on both voltage measurment channels AI0 and AI1, one is for the reference sensor voltage, V_r,measurement, and the other is for the check standand output viltage, V_u. The standard uncertainty of the voltage ratio is listed below:

$$u(V_{Vol,trace}) = \sqrt{u_{\text{AI0}}(V_R)^2 + u_{\text{AI1}}(V_R)^2} \quad (9)$$

where $u_{\text{AI0}}(V_R)$ *and* $u_{\text{AI1}}(V_R)$ are standard uncertainties from the calibration report for each channel under difference voltage amplitudes and frquencies. Here, for every calibrated channel x,

$$u_{AIx}(V_R) = V_{\text{mean},x} \times \frac{U_{Vol\_trace}}{K} \quad (10)$$

where $V_{\text{mean},x}$ is the voltage input amplitude for each channel calibrated
U_{Vol_trace} is the relative expanded uncertainty from calibration report [7], for frequeny range from 10 Hz to 10 kHz and the voltage range from 10 mV to 1 V, the relative expanded uncertainty is 0.38 %.[7]
K is the coverage factor, and K=2

(iii) Relative standard uncertainty of shaker transverse and rocking motion

The transverse and rocking motion, which is the acceleration deviating from the intended excitor moving direction, will vary with the mounting orientation of the reference and test sensor, the exciting amplitude, and the loading on the shaker with different frequency. This contributor is not considered as random because the influence of the transverse motion and rocking motion remains mostly consistent for check standard is mounted on the excitor.

The transverse and rocking motion from shaker is vector addition of the two lateral axes (i.e. x-axis and y-axis in this case) normalized by the primary direction(z-axis). It can be determined using a tri-axial sensor mounted on the reference sensor surface as shown in Fig. 2.

Firstly, the frequency response function of acceleration level was performed on both normal axes ,a_x and a_y referring to the acceleration on the paralle axis, a_z, which is the expected exciting (calibration) direction. The rocking and transverse acceleration level on two normal direction, a_x and a_y, were determined individually by following expressions:

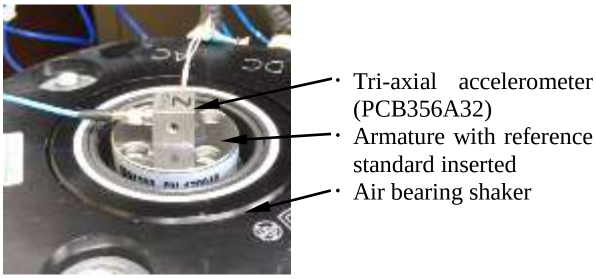

Fig.2 Shaker's transverse and rocking motion measurement setup

$$a_x = a_z \times G_{a(xz)},\ \ a_y = a_z \times G_{a(yz)} \tag{11}$$

where a_z is the acceleration level on the parallel direction and Ga(xz) and Ga(yz) are the gains of acceleration amplitude on x-axis and y-axis referring to z-axis.

Then rocking and transverse motion during the test could be calculated as following equation

$$a_T = \sqrt{a_x^2 + a_y^2} \tag{12}$$

For corresponded the inherently nature of the rocking motion. We measured the relatived motions on the normal axes referring to parallel axis rather than the actual acceleration levels.

Uncertainty from transverse and rocking motion from shaker, the rocking and transverse error motion contribute to the vlotage rading ,$V(_{R\_Rocking})$, is calculated as follows:

$$V_{R\_Rocking} = \sqrt{(S_{v,1}^2 + S_{v,2}^2) \times a_\mathrm{T}^2} \tag{13}$$

In Eq.(13), $S_{v,1}$ and $S_{v,2}$ are the nominal transverse sensitivities of reference sensor and test sensor, respectively. a_T is the measured transverse and rocking motion of the excitor.

The transverse sensitivity specification for both refernce sensor and the check standard is specified as 5 %[8,9]. The standard deviation of the shaker cross-axis and rocking motion component uncertainty is shown in Fig. 3. The relative standard uncertainty from the transverse and rocking motion, $u(V_{R\_Rocking})$, could be determined by Eq. (14).

$$u(V_{R\_Rocking}) = \frac{V_{R\_Rocking}}{\sqrt{18}} \tag{14}$$

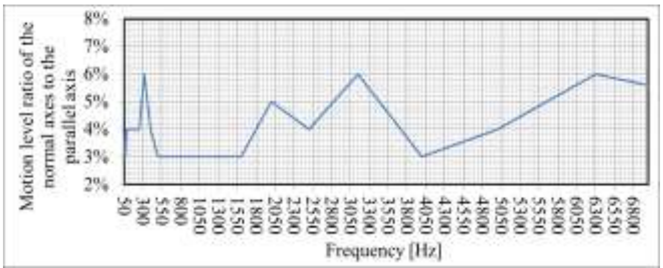

Fig. 3 Results of transverse and rocking motion

(iv) Relative standard uncertainty of sensitivity drift on the reference standard

The drift of the reference accelerometer is systemic in nature, it is typically characterized by random component. Referring to the accelerometer specfication[8], the stability of the reference standard is estimated to be 0.05 % per year. For two yeas calibration onterval the worst drift is 0.1 %, and here we used 0.2 % for the sensitivity shift, S_{Drift}.

3.4 Covariance estimation

The assumption in this system validation is based on that each of measurement input is not correlated. Thus covariance between different measurement inputs is negligible.

3.5 Combined standard uncertainty

The combined standard uncertainty of accelerometer sensitivity S_u by comparison method is concluded as:

$$u_c(S_u) = \sqrt{\left[\left(\frac{\partial S_u}{\partial S_r}\right)^2 u^2(S_r) + \left(\frac{\partial S_u}{\partial V_R}\right)^2 u^2(V_R)\right]} \tag{15}$$

After substituting the appropriate evaluated values into the above uncertainty equation, then calculated results of $U_c(S_u)$ are listed in Table 4 and Table 5.

3.6 Expanded uncertainty

The expanded uncertainty is obtained from multiplying the combined standard uncertainty by the associated coverage factor *k* at a required confidence level (presently 95 % adopted in the system). The value of coverage factor *k* at the 95 % confidence level is then found from the Student-*t* distribution with a known effective degrees of freedom, ν_eff. The *effective* degrees of freedom is approximated by the Welch-Satterthwaite equation shown as follows:

$$\nu_\mathrm{eff} = \frac{u_c^4(S_u)}{\sum \frac{\left(\frac{\partial S_u}{\partial x_i}\right)^4 u^4(x_i)}{\nu_i}} \tag{16}$$

where $u_c(S_u)$ is combined standard uncertainty of S_u

$u(X_i)$ is the standard uncertainty of each input measurand

ν_i is the degrees of freedom associated with each uncertainty component which can be estimated shown in the following way

The degrees of freedom of type A uncertainty is ν_i=n-1 with *n* repetitions of measurement. The degrees of freedom of type B uncertainty may be approximated as $\nu_i \approx (1/2)(R)^{-2}$, in which R is reliability of the uncertainty and expressed as (R ≈ 1 – confidence level) in this document. For 95 % confidence level, R = 5 %. Thus, ν_i is 200 for each uncertainty source of related instrument, and ν_eff will be obtained by substituting each $u(X_i)$ and associated ν_i into the Welch-Satterthwaite equation.

4. CONCLUSION

The system configuration and the uncertainty estimation method through type A and Type B uncertainties are introduced for calculating the measurement uncertainty of accelerometers' voltage sensitivity and effective degrees of freedom with a chosen coverage factor. The calibration station is for conducting the accelerometer sensitivity with frequency range from 50 Hz to 7 kHz. The assessment on the best measurement capability of the calibration station was perform using the uncertainties evaluation for calibration performed using a check standard. The results of the relative expaneded uncertainty for the voltage sensitivities calibration for accelerometer are less than 1.4 %, 1.9 % and 2.9 % within frequency range from 50 Hz to 1.5 kHz, from 1.5 kHz to 5 kHz, and from 5 kHz to 7 kHz, respectively.

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Table 4 Uncertainty components for accelerometer's voltage sensitivity magnitude determination

Uncertainty Component, x_i	Probability distribution Type	Probability distribution Factor	Type	Relative Standard uncertainty $u(x_i)$	Sensitivity coefficients c_i	Degree of freedom ν_i
Voltage Error, V_{Vol_Trace}	Rectangular	$1/\sqrt{3}$	B	$\sqrt{u_{AI0}(V_R)^2 + u_{AI1}(V_R)^2}$	S_r	200
Reference standard Calibration, S_{Trace}	Normal (k=2)	1/2	B	$S_r \times (U_{sen\_trace}/2)$	S_r	200
Reference standard sensivity drift, S_{Drift}	Rectangular	$1/\sqrt{3}$	B	$0.2/\sqrt{3}$	S_r	50
Shaker transverse and rocking motion, $V_{R\_Rocking}$	Special	$1/\sqrt{18}$	B	$\sqrt{(S_{v,1}^2 + S_{v,2}^2) \times a_T^2/\sqrt{18}}$	S_r	12.5
The long term repetition, $V_{SDV\_Rep\_1}$	t	$1/\sqrt{3}$	A	$V_{SDV\_Rep\_1}/\sqrt{3}$	V_R	38
The short term repetition, $V_{SDV\_Rep\_2}$	t	$1/\sqrt{3}$	A	$V_{SDV\_Rep\_2}/\sqrt{3}$	V_R	4

Table 5 Uncertainty Budgets for accelerometer sensitivity calibration from 50 Hz to 7 kHz

		Frequency Range [Hz]		
		50 to 1.5k	1.5k to 5k	5k to 7k
Relative uncertainty Component,	Voltage Error	0.30	0.30	0.30
	Reference standard Calibration	0.58	0.86	1.13
	Reference standard sensivity drift	0.07	0.07	0.07
	Shaker transverse and rocking motion	0.06	0.08	0.11
	The long term repetition	0.04	0.12	0.24
	The short term repetition	0.01	0.01	0.01
	Relative combined standard uncertainty, $u_c(S_r)$	0.66	0.92	1.20
	Effective degree of freedom, ν_{eff}	306.3	258.7	248.3
	Coverage factor, k	1.97	1.97	1.97
	Relative expanded uncertainty, U [%]	1.4	1.9	2.4